TME-EMT-Wiki

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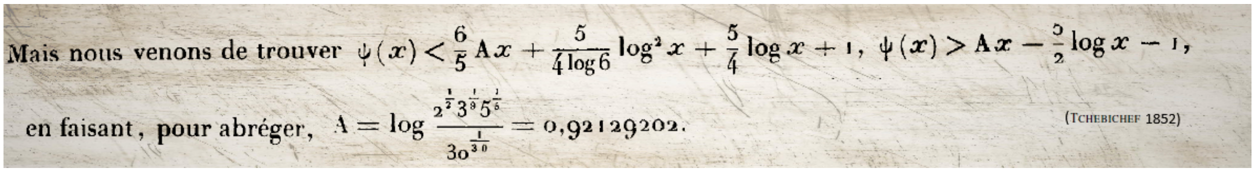
Contents


Mais nous venons de trouver $\psi(x) < \frac{6}{5}Ax + \frac{5}{4\log 6}\log^2 x + \frac{5}{4}\log x + 1$, $\psi(x) > Ax - \frac{5}{2}\log x - 1$,

en faisant, pour abréger, $A = \log \frac{2^{\frac{1}{2}}3^{\frac{1}{3}}5^{\frac{1}{5}}}{30^{\frac{1}{30}}} = 0{,}92129202.$

(Tchebichef 1852)

Fig. 1: Fully explicit results in multiplicative number theory are often scattered through the litterature. The aim of this site is to present annoted bibliographies in order to keep track of the current knowledge. By the way, the acronym TME-EMT stands for

Théorie Multiplicative Explicite des nombres / Explicit Multiplicative number Theory

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Note

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1 Explicit bounds on primes

Collecting references: [Dusart, 1998], [Dusart, 2018]

1.1 1. Bounds on primes, in special ranges

The paper [Rosser and Schoenfeld, 1962], contains several bounds valid only when the variable is small enough. These are proved by direction verifications. Different direct computations can be used to get effect in limited range. Here is a typical result.

Theorem ([Büthe, 2016])

Assume the Riemann Hypothesis has been checked up to height H_0. Then when x satisfies $\sqrt{x/\log x} \le H_0/4.92$, we have

- $|\psi(x) - x| \le \frac{\sqrt{x}}{8\pi} \log^2 x$ when $x > 59$,
- $|\theta(x) - x| \le \frac{\sqrt{x}}{8\pi} \log^2 x$ when $x > 599$,
- $|\pi(x) - \mathrm{li}(x)| \le \frac{\sqrt{x}}{8\pi} \log x$ when $x > 2657$.

If we use the value $H_0 = 3 \cdot 10^{12}$ obtained by Platt and Trudgian in [Platt and Trudgian, 2021] (which improves previous results in [Platt, 2017]), these bounds are thus valid for $x \le 2.16 \cdot 10^{25}$. In [Büthe, 2018] the following bounds are also obtained.

Theorem (2018)

We have

- $|\psi(x) - x| \le 0.94\sqrt{x}$ when $11 < x \le 10^{19}$,
- $0 < \mathrm{li}(x) - \pi(x) \le \frac{\sqrt{x}}{\log x}\left(1.95 + \frac{3.9}{\log x} + \frac{19.5}{\log^2 x}\right)$ when $2 \le x \le 10^{19}$.

1.2 2. Bounds on primes, without any congruence condition

The subject really started with the four papers [Rosser, 1941], [Rosser and Schoenfeld, 1962], [Rosser and Schoenfeld, 1975] and [Schoenfeld, 1976].

We recall the usual notation: $\pi(x)$ is the number of primes up to x (so that $\pi(3) = 2$), the function $\psi(x)$ is the summatory function of the van Mangold function Λ, i.e. $\psi(x) = \sum_{n\le x}\Lambda(n)$, while we also define $\vartheta(x) = \sum_{p\le x}\log p$. Here are some elegant bounds that one can find in those papers.

Theorem (1962)

- For $x > 0$, we have $\psi(x) \le 1.03883x$ and the maximum of $\psi(x)/x$ is attained at $x = 113$.
- When $x \ge 17$, we have $\pi(x) > x/\log x$.
- When $x > 1$, we have $\displaystyle\sum_{p\le x} 1/p > \log\log x$.
- When $x > 1$, we have $\displaystyle\sum_{p\le x} (\log p)/p < \log x$.

There are many other results in those papers. In [Dusart, 1999] , one can find among other things the inequality

and also

Theorem (1999)

- When $x \ge e^{22}$, we have $\psi(x) = x + O^*\left(0.006409\dfrac{x}{\log x}\right)$.
- When $x \ge 10\,544\,111$, we have $\vartheta(x) = x + O^*\left(0.006788\dfrac{x}{\log x}\right)$.
- When $x \ge 3\,594\,641$, we have $\vartheta(x) = x + O^*\left(0.2\dfrac{x}{\log^2 x}\right)$.
- When $x > 1$, we have $\vartheta(x) = x + O^*\left(515\dfrac{x}{\log^3 x}\right)$.

This is improved in [Dusart, 2018] , and in particular, it is shown that the 515 above can be replaced by 20.83 and also that

Bounds of the shape $|\psi(x) - x| \le \epsilon x$ have started appearing in [Rosser and Schoenfeld, 1962] The latest paper is [Faber and Kadiri, 2015] with its corrigendum [Faber and Kadiri, 2018], where the explicit density estimate from [Kadiri, 2013] is put to contribution, even for moderate values of the variable. In particular, when $x \geq 485\,165\,196$, we have

Sharper estimates of this type for larger x were obtained in [] and [Johnston and Yang, 2023]. In particular, we have [Johnston and Yang, 2023]

In [Platt and Trudgian, 2021] , one find the following estimate

Theorem (2021)

When $x \geq e^{2000}$, we have $\left|\frac{\psi(x)-x}{x}\right| \leq 235\,(\log x)^{0.52}\exp -\sqrt{\frac{\log x}{5.573412}}$.

The paper [Johnston and Yang, 2023] further proves in Theorem 1.1 the next estimate.

Theorem (2023)

When $x \geq 2$, we have $\left|\frac{\psi(x)-x}{x}\right| \leq 9.39\,(\log x)^{1.51}\exp -0.8274\sqrt{\log x}$.

When $x \geq 23$, we have $\left|\frac{\psi(x)-x}{x}\right| \leq 0.026\,(\log x)^{1.801}\exp -0.1853\frac{(\log x)^{3/5}}{(\log\log x)^{1/5}}$.

Several estimates are also provided for $\psi(x)$, $\pi(x)$ and $\vartheta(x)$.

A lighly sharper estimate than that above was obtained at the same time in []

Theorem (2023)

When $x \geq 2$, we have $\left|\frac{\psi(x)-x}{x}\right| \leq 9.22022\,(\log x)^{1.5}\exp -0.8476836\sqrt{\log x}$.

Refined bounds for $\pi(x)$ are to be found in [Panaitopol, 2000] and in [Axler, 2016]

By comparing in an efficient manner with $\psi(x) - x$, [Ramaré, 2013], obtained the next two results. [There was an error in this paper which is corrected below].

Theorem (2013)

For $x > 1$, we have $\sum_{n\leq x}\Lambda(n)/n = \log x - \gamma + O^*(1.833/\log^2 x)$. When $x \geq 1\,520\,000$, we can replace the error term by $O^*(0.0067/\log x)$. When $x \geq 468\,000$, we can replace the error term by $O^*(0.01/\log x)$. Furthermore, when $1 \leq x \leq 10^{10}$, this error term can be replaced by $O^*(1.31/\sqrt{x})$.

Theorem (2013)

For $x \geq 8950$, we have $\displaystyle\sum_{n\leq x}\Lambda(n)/n = \log x - \gamma + \frac{\psi(x)-x}{x} + O^*\Big(\frac{1}{2\sqrt{x}} + 1.75\cdot 10^{-12}\Big)$.

[Mawia, 2017] developed the method to incorporate more functions (and corrected the initial work), extending results of [Rosser and Schoenfeld, 1962] . Here are some of his results.

Theorem (2017)

For $x \geq 2$, we have $\sum_{p \leq x} \frac{1}{p} = \log\log x + B + O^*\Big(\frac{4}{\log^3 x}\Big)$. When $x \geq 1000$, one can replace the 4 in the error term by 2.3, and when $x \geq 24284$, by 1. The constant $B \approx 0.261497$ is known as the Meissel-Mertens constant.

Theorem (2017)

For $\log x \geq 4635$, we have $\sum_{p \leq x} \frac{1}{p} = \log\log x + B + O^*\Big(1.1 \frac{\exp(-\sqrt{0.175 \log x})}{(\log x)^{3/4}}\Big)$.

When truncating sums over primes, Lemma 3.2 of [Ramaré, 2016] is handy.

Theorem (2016)

Let f be a C^1 non-negative, non-increasing function over $[P, \infty)$, where $P \geq 3\,600\,000$ is a real number and such that $\lim_{t \to \infty} t f(t) = 0$. We have

$\sum_{p \geq P} f(p) \log p \leq (1+\epsilon) \int_P^\infty f(t) dt + \epsilon P f(P) + P f(P)/(5 \log^2 P)$

with $\epsilon = 1/914$. When we can only ensure $P \geq 2$, then a similar inequality holds, simply replacing the last $1/5$ by 4.

The above result relies on (5.1*) of [Schoenfeld, 1976] because it is easily accessible. However on using Proposition 5.1 of [Dusart, 2016], one has access to $\epsilon = 1/36260$.

Here is a result due to [Treviño, 2012].

Theorem (2012)

For x a positive real number. If $x \geq x_0$ then there exist c_1 and c_2 depending on x_0 such that $\frac{x^2}{2\log x} + \frac{c_1 x^2}{\log^2 x} \leq \sum_{p \leq x} p \leq \frac{x^2}{2\log x} + \frac{c_2 x^2}{\log^2 x}$. The constants c_1 and c_2 are given for various values of x_0 in the next table.

x_0	c_1	c_2
315437	0.205448	0.330479
468577	0.211359	0.32593
486377	0.212904	0.325537
644123	0.21429	0.322609
678407	0.214931	0.322326
758231	0.215541	0.321504
758711	0.215939	0.321489
10544111	0.239818	0.29251

Further estimates can be found in [Axler, 2019] (Proposition 2.7 and Corollary 2.8). In

[Deléglise and Nicolas, 2019] (Proposition 2.7 and Corollary 2.8) and [Deléglise and Nicolas, 2019] (Proposition 2.5, Corollary 2.6, 2.7 and 2.8), we find among other results the next two.

Theorem (2019)

On setting $\pi_r(x) = \sum_{p\le x} p^r$, we have

$$\frac{3x^2}{20(\log x)^4} \le \pi_1(x) - \left(\frac{x^2}{2\log x} + \frac{x^2}{4(\log x)^2} \frac{x^2}{4(\log x)^3}\right) \le \frac{107x^2}{160(\log x)^4}$$

where the upper estimate is valid when $x \ge 110117910$ and the lower one when $x \ge 905238547$.

We have

$$\frac{-1069x^3}{648(\log x)^4} \le \pi_2(x) - \left(\frac{x^3}{3\log x} + \frac{x^3}{9(\log x)^2} \frac{x^3}{27(\log x)^3}\right) \le \frac{11181x^3}{648(\log x)^4}$$

where the upper estimate is valid when $x \ge 60173$ and the lower one when $x \ge 1091239$. In addition,

$$\pi_3(x) \le 0.271\frac{x^4}{\log x} \quad \text{for } x \ge 664,$$

$$\pi_4(x) \le 0.237\frac{x^5}{\log x} \quad \text{for } x \ge 200,$$

$$\pi_5(x) \le 0.226\frac{x^5}{\log x} \quad \text{for } x \ge 44,$$

For $x \ge 1$ and $r \ge 5$, we have $\pi_r(x) \le \dfrac{\log 3}{3}\left(1 + (2/3)^r\right)\dfrac{x^{r+1}}{\log x}$.

1.3 3. Bounds containing the n-th prime

Denote by p_n the n-th prime. Thus $p_1 = 2,\ p_2 = 3,\ p_4 = 5, \cdots$.

The classical form of the prime number theorem yields easily $p_n \sim n\log n$. [Rosser, 1938] shows that this equivalence does not oscillate by proving that p_n is greater than $n\log n$ for $n \ge 2$.

The asymptotic formula for p_n can be developped as shown in [Cipolla, 1902]

This asymptotic expansion is the inverse of the logarithmic integral Li(x) obtained by series reversion.

[Rosser, 1938] obtained a finite version of the above: for every $n > 1$:

He improves these results in [Rosser, 1941] : for every $n \ge 55$,

This result was subsequently improved by Rosser and Schoenfeld [Rosser and Schoenfeld, 1962] in 1962 to

where the first and the second inequalities holds for $n > 1$ and $n > 19$, respectively. Those constants were subsequently reduced by [Robin, 1983] . In particular, the lower bound

is valid for $n > 1$ and the constant 1.0072629 can be replaced by 1 for $p_k \le 10^{11}$. Then [Massias and Robin, 1996] showed that the constant term equals to 1 in the lower bound was admissible for $p_k < e^{598}$ and $p_k > e^{1800}$. Finally, [Dusart, 1999] showed that for all $n > 1$ and also that

for $n > 39017$ i.e. $p_n > 467473$.

In [Carneiro *et al.*, 2019], the authors prove the next result.

Theorem (2019)

Under the Riemann Hypothesis, for all $n \geq 3$,we have, $p_{n+1} - p_n \leq \frac{22}{25}\sqrt{p_n}\log p_n$.

In [Axler, 2019] we find (Theorem 1.6 and 1.7) the next result.

Theorem (2019)

We have $\sum_{i\leq k} p_i \geq \frac{k^2}{4} + \frac{k^2}{4\log k} - \frac{k^2(\log\log k - 2.9)}{4(\log k)^2} \quad (\text{for } k \geq 6309751),$

as well as

$\sum_{i\leq k} p_i \leq \frac{k^2}{4} + \frac{k^2}{4\log k} - \frac{k^2(\log\log k - 4.42)}{4(\log k)^2} \quad (\text{for } k \geq 256376).$

In [De Koninck and Letendre, 2020] we find in passing (Lemma 4.8) the next result.

Theorem (2020)

We have $\sum_{i\leq k} \log p_i \leq k\big(\log k + \log\log k - 1/2\big) \quad (\text{for } k \geq 5)$, as well as $\sum_{i\leq k} \log\log p_i \geq k\left(\log\log k + \frac{\log\log -3/2}{\log k}\right) \quad (\text{for } k \geq 6).$

1.4 4. Bounds on primes in arithmetic progressions

Collecting references: [McCurley, 1984], [McCurley, 1984], [Ramaré and Rumely, 1996], [Dusart, 2002]

Lemma 10 of [Moree, 2004] , section 4 of [Moree and te Riele, 2004] .

A consequence of Theorem 1.1 and 1.2 of [Bennett *et al.*, 2018] states that

Theorem (2018)

We have

$$\max_{3\leq q\leq 10^4} \max_{x\geq 8\cdot 10^9} \max_{\substack{1\leq a\leq q,\\ (a,q)=1}} \frac{\log x}{x}\Big|\sum_{\substack{n\leq x,\\ n\equiv a[q]}} \Lambda(n) - \frac{x}{\varphi(q)}\Big| \leq 1/840.$$

When $q \in (10^4, 10^5]$, we may replace $1/840$ by $1/160$ and when $q \geq 10^5$, we may replace $1/840$ by $\exp(0.03\sqrt{q}\log^3 q)$.

Furthermore, we may replace $\sum_{\substack{n\le x,\\ n\equiv a[q]}}\Lambda(n)$ by $\sum_{\substack{p\le x,\\ p\equiv a[q]}}\log p$ with no changes in the constants.

Similarly, as a consequence of Theorem 1.3 of [Bennett *et al.*, 2018] states that

Theorem (2018)

We have

$$\max_{3\le q\le 10^4}\max_{x\ge 8\cdot 10^9}\max_{\substack{1\le a\le q,\\ (a,q)=1}}\frac{\log^2 x}{x}\Big|\sum_{\substack{p\le x,\\ p\equiv a[q]}}1-\frac{\mathrm{Li}(x)}{\varphi(q)}\Big|\le 1/840.$$

When $q\in(10^4,10^5]$, we may replace $1/840$ by $1/160$ and when $q\ge 10^5$, we may replace $1/840$ by $\exp(0.03\sqrt{q}\log^3 q)$.

Concerning estimates with a logarithmic density, in [Ramaré, 2009] and in [Platt and Ramaré, 2017] estimates for the functions $\sum_{\substack{n\le x,\\ n\equiv a[q]}}\Lambda(n)/n$ are considered. Extending computations from the former, the latter paper Theorem 8.1 reads as follows.

Theorem (2016)

We have $\max_{q\le 1000}\max_{q\le x\le 10^5}\max_{\substack{1\le a\le q,\\ (a,q)=1}}\sqrt{x}\Big|\sum_{\substack{n\le x,\\ n\equiv a[q]}}\frac{\Lambda(n)}{n}-\frac{\log x}{\varphi(q)}-C(q,a)\Big|\in(0.8533,0.8534)$ and the maximum is attained with $q=17$, $x=606$ and $a=2$.

The constant $C(q,a)$ is the one expected, i.e. such that $\sum_{\substack{n\le x,\\ n\equiv a[q]}}\frac{\Lambda(n)}{n}-\frac{\log x}{\varphi(q)}-C(q,a)$ goes to zero when x goes to infinity.

When q belongs to "Rumely's list", i.e. in one of the following set:

- $\{k\le 72\}$
- $\{k\le 112, k \text{ non premier}\}$
- $\{116,117,120,121,124,125,128,132,140,143,144,156,163,$
  $169,180,216,243,256,360,420,432\}$

Theorem 2 of [Ramaré, 2009] gives the following.

Theorem (2002)

When q belongs to Rumely's list and a is prime to q, we have $\sum_{\substack{n\le x,\\ n\equiv a[q]}}\frac{\Lambda(n)}{n}=\frac{\log x}{\varphi(q)}+C(q,a)+O^*(1)$ as soon as $x\ge 1$.

More precise bounds are given.

1.5 5. Least prime verifying a condition

In [Bach and Sorenson, 1996] , the authors estimate the magnitude of the first prime number in reduced residue classes.

Theorem (1996)

Let m and q be integers, with $gcd(m;q) = 1$. There is a prime $p \equiv m(\mod q)$ satisfying $p < 2(q\log q)^2$.

We also have the following result connected to prime gaps in residue classes [Kadiri, 2008].

Theorem (2008)

Let $q \geq 3$ be a non-exceptional modulus and let $(a, q) = 1$. For any $\epsilon > 0$, there exists a computable constant $\alpha > 0$ such that, if $x \geq \alpha\log^2 q$, then the interval $[e^x, e^{x+\epsilon}]$ contains a prime $p \equiv a(\mod q)$.

The constant α in the theorem above was computed for several ϵ and $q \geq q_0$ as in the following table

2

Explicit bounds on the Moebius function

Collecting references: [Diamond and Erdös, 1980], [Deléglise and Rivat, 1996], [Borwein *et al.*, 2008].

2.1 1. Bounds on $M(D) = \sum_{d \le D} \mu(d)$

The first explicit estimate for $M(D)$ is due to [von Sterneck, 1898] where the author proved that $|M(D)| \le \frac{1}{9}D + 8$ for any $D \ge 0$. Here is a first popular estimate.

Theorem ([Mac Leod, 1967])

When $D \ge 0$, we have $|M(D)| \le \frac{1}{80}D + 5$. When $D \ge 1119$, we have $|M(D)| \le D/80$.

We mention at this level the annoted bibliography contained at the end of [Dress, 1983/84].

Theorem ([Costa Pereira, 1989])

When $D \ge 120\,727$, we have $|M(D)| \le D/1036$.

Improving on this method, the next result was obtained.

Theorem ([Dress and El Marraki, 1993])

When $D \ge 617\,973$, we have $|M(D)| \le D/2360$.

One of the arguments is the next estimate.

Theorem ([Dress, 1993])

When $33 \leq D \leq 10^{12}$, we have $|M(D)| \leq 0.571\sqrt{D}$.

This has been extended by [Kotnik and van de Lune, 2004] to 10^{14} and recently by Hurst.

Theorem ([Hurst, 2018])

When $33 \leq D \leq 10^{16}$, we have $|M(D)| \leq 0.571\sqrt{D}$.

Another tool is given in [Cohen and Dress, 1988], where refined explicit estimates for the remainder term of the counting functions of the squarefree numbers in intervals are obtained. The latest best estimate of this shape comes from [Cohen *et al.*, 1996]. This preprint being difficult to get, it has been republished in [Cohen *et al.*, 2007].

Theorem (1996)

When $D \geq 2\,160\,535$, we have $|M(D)| \leq D/4345$.

These results are used in [Dress, 1999] to study the discrepancy of the Farey series.

Concerning upper bounds that tend to 0, [Schoenfeld, 1969] was the pioneer and obtained, among other things, the following estimates.

Theorem (1969)

When $D > 0$, we have $|M(D)|/D \leq 2.9/\log D$.

Those were later improved in:

Theorem ([El Marraki, 1995])

When $D \geq 685$, we have $|M(D)|/D \leq 0.10917/\log D$.

Ramaré further improved those bounds for larger D.

Theorem ([Ramaré, 2013])

When $D \geq 1\,100\,000$, we have $|M(D)|/D \leq 0.013/\log D$.

Some bounds including coprimality conditions were also obtained.
For instance, we have

Theorem ([Ramaré, 2015])

When $1 \le q < D$, we have $\Big| \sum_{\substack{d \le D, \\ (d,q)=1}} \mu(d) \Big| / D \le \frac{q}{\varphi(q)} / (1 + \log(D/q))$.

Theorem ([Ramaré, 2015])

For $1 \le q < D$, we have $\frac{\varphi(q)}{q} \log(D/q) \Big| \sum_{\substack{d \le D, \\ (d,q)=1}} \mu(j) \Big| / D \le \begin{cases} 0.997, & D/q > 1, \\ 0.429, & D/q \ge 490, \\ 1/5, & D/q \ge 4536, \\ 0.0918, & D/q \ge 48513. \end{cases}$.

The best uniform bound (in q) of the form above for $D/q > 1$ were obtained in [de Camargo, 2025].

Theorem ([de Camargo, 2025])

When $1 \le q < D$, we have $\frac{\varphi(q)}{q} \log(D/q) \Big| \sum_{\substack{d \le D, \\ (d,q)=1}} \mu(d) \Big| / D \le \begin{cases} 0.3131, & q = 2, \\ 0.2663, & q = 3, \\ 0.2335, & q = 5 \\ 0.1738, & q = 6, \\ 0.2102, & q \ge 7. \end{cases}$. These constants are optimal up to the third decimal place.

2.2 2. Bounds on $m(D) = \sum_{d \le D} \frac{\mu(d)}{d}$

[Mac Leod, 1967] shows that the sum $m(D)$ takes its minimal value at $D = 13$. A folklore result reads as follows.

Theorem ([Granville and Ramaré, 1996])

When $D \ge 0$ and for any integer $r \ge 1$, we have $\Big| \sum_{\substack{d \le D, \\ (d,r)=1}} \frac{\mu(d)}{d} \Big| \le 1$.

In fact, Lemma 1 of [Davenport, 1937] already contains the requisite material. The inequality in the theorem above was rediscovered and generalized in [Tao, 2010] to sums over semi-groups generated by arbitrary sets of prime numbers. Further refinements of the result above were obtained for larger D as shown below.

Theorem ([Ramaré, 2015])

When $D \ge 7$, we have $|\sum_{d \le D} \mu(d)/d| \le 1/10$. We can replace the couple (7, 1/10) by (41, 1/20) or (694, 1/100).

This result has been further extended.

Theorem ([Ramaré, 2013])

When $D \geq 0$ and for any integer $r \geq 1$ and any real number $\varepsilon \geq 0$, we have $\left|\sum_{\substack{d\leq D,\\(d,r)=1}} \mu(d)/d^{1+\varepsilon}\right| \leq 1+\varepsilon$.

Concerning upper bounds that tend to 0, here is a first estimate.

Theorem ([El Marraki, 1996])

When $D \geq 33$ we have $|m(D)| \leq 0.2185/\log D$.

When $D > 1$ we have $|m(D)| \leq 726/(\log D)^2$.

The second bound above was improved:

Theorem ([Bordellès, 2015])

When $D > 1$ we have $|m(D)| \leq 546/(\log D)^2$.

[Ramaré, 2013] proves several bounds of the shape $m(D) \ll 1/\log D$. Those results were improved using the tools of [Balazard, 2012], which provide us with a better manner to convert bounds on $M(D)$ into bounds for $m(D)$. Here is one result obtained.

Theorem ([Ramaré, 2015])

When $D \geq 463\,421$ we have $|m(D)| \leq 0.0144/\log D$.

We can, for instance, replace the couple (463 421, 0.0144)by any of (96 955, 1/69), (60 298, 1/65), (1426, 1/20) or (687, 1/12).

In [Ramaré, 2014] and [Ramaré, 2015], the problem of adding coprimality conditions is further addressed. Here is one of the results obtained.

Theorem (2015)

When $1 \leq q < D$ we have $\left|\sum_{\substack{d\leq D,\\(d,q)=1}} \mu(d)/d\right| \leq \frac{q}{\varphi(q)}0.78/\log(D/q)$. When $D/q \geq 24233$, we can replace 0.78 by 17/125.

The estimates above were improved in 2025.

Theorem ([de Camargo, 2025])

When $1 \leq q < D$ we have $\left|\sum_{\substack{d\leq D,\\(d,q)=1}} \mu(d)/d\right| \leq \frac{q}{\varphi(q)} 0.3055/\log(D/q)$.

2.3 3. Bounds on $\check{m}(D) = \sum_{d\leq D} \mu(d)\log(D/d)/d$

The initial investigations on this function go back to [Daublebsky von Sterneck, 1902].

Theorem ([Ramaré, 2015])

When $3846 \leq D$ we have $|\check{m}(D) - 1| \leq 0.00257/\log D$. When $D > 1$, we have $|\check{m}(D) - 1| \leq 0.213/\log D$.

This implies in particular that

Theorem (2015)

When $222 \leq D$ we have $|\check{m}(D) - 1| \leq 1/1250$. When $D > 1$, the optimal bound 1 holds.

These bounds are a consequence of the identity:
$|\check{m}(D) - 1| \leq \frac{\frac{7}{4} - \gamma}{D^2} \int_1^D |M(t)| dt + \frac{2}{D}$. It is also proved that, for any $D \geq 1$, we have

2.4 4. Miscellanae

Here is an elegant wide ranging estimate, taken from Claim 3.1 of the referenced paper.

Theorem ([Treviño, 2015])

For real $D \geq 1$ we have $|\sum_{d>D} \mu(d)/d^2| \leq 1/D$.

Taking the limit for $D \to K + 1$ in the inequality above and after replacing $K + 1$ by D, we get $|\sum_{d\geq D} \mu(d)/d^2| \leq 1/D$, which improves upon the trivial upper bound $1/(D-1)$.

2.5 5. The Moebius function and arithmetic progressions

The results in this section are scarce.

Theorem ([Bordellès, 2015])

Let χ be a non-principal Dirichlet character modulo $q \geq 37$ and let $k \geq 1$ be some integer. Then $\left| \sum_{\substack{n \leq x, \\ (n,k)=1}} \frac{\mu(n)\chi(n)}{n} \right| \leq \frac{k}{\varphi(k)} \frac{2\sqrt{q}\log q}{L(1,\chi)}$.

3

Exact computations of the number of primes

Collecting references: [Deléglise and Rivat, 1998], [Platt, 2011].

Papers [Deléglise and Rivat, 1996] and Silva, 2006 present tables of values of $\pi(x)$ for powers of 10 up to 10^{18} and 10^{22}, respectively. The later also computed $\pi(x)$ for $x = 2^k$ for $k \leq 72$. [Platt, 2011] also computed $\pi(x)$ for $x = 10^{23}$. In Deléglise et al, 2004, the authors computes the functions $\pi(x, 4, 1)$ and $\pi(x, 4, 3)$ that counts the number of prime numbers up to x in the residue classes $p \equiv 1 \mod 4$ and $p \equiv 3 \mod 4$, respectively, for $x = m \times 10^k$ for $m = 1, 2, \ldots, 9$ and $k = 10, 11, \ldots, 20$ (for $k = 20$, only $m = 1$ was considered).

Paper [Deléglise and Rivat, 1996] computes the Chebyshev function $\psi(x)$ for $x = m \times 10^k$ for $m = 1, 2, \ldots, 9$ and $k = 6, 7, \ldots, 15$ (for $k = 15$, only $m = 1$ was considered).

4 Computations of arithmetical constants

Collecting references: [Cazaran and Moree, 1999].

4.1 1. Euler products of rational functions

The computation of Euler product of rational function is dealt with in [Moree, 2000]. The reader may also consult the following web pagefootnote{url{http://guests.mpim-bonn.mpg.de/moree/Moree.en.html}}.

4.2 2. Some special sums over prime values that are derivatives

5

Explicit results on exponential sums

Collecting references: [Daboussi and Rivat, 2001].

5.1 1. Bounds with the first derivative

We start with the Kusmin-Landau Lemma.

Theorem

Let f be a function over $[a,b]$ such that f' is monotonic and satisfies $\theta \le f'(u) \le 1-\theta$ for some $\theta \in (0,1/2]$. Then $\left|\sum_{a\le n\le b} e(f(n))\right| \le \cot\frac{\pi\theta}{2} \le \frac{2}{\pi\theta}$.

5.2 2. Bounds with the second derivative

Here is a corrected version of Lemma 3 of [Cheng and Graham, 2004], see Lemma 2.3 of [Patel, 2022].

Theorem (2004)

Let f be a real-valued function with two continuous derivatives on $[N+1, N+L]$. Suppose there are $W>1$ and $\lambda>1$ such that $1 \le W|f''(x)| \le \lambda$ for every $x \in [N+1, N+L]$. Then we have $\left|\sum_{n=N+1}^{N+L} \exp(2i\pi f(n))\right| \le 2\left(\frac{L\lambda}{W}+2\right)\left(2\sqrt{\frac{W}{\pi}}+1\right)$.

5.3 3. Bounds with the third derivative

Here is Lemma 1.2 of

[Hiary, 2016]. See Lemma 2.3 [Patel, 2022].

Theorem (2016)

Let f be a real-valued function with three continuous derivatives on $[N+1, N+L]$. Suppose there are $W > 1$ and $\lambda > 1$ such that $1 \leq W|f'''(x)| \leq \lambda$ for every $xin [N+1, N+L]:math:. *Then, for any `eta > 0$, we have :math:*displaystyle biggl|sum_{n= N+1}^{N+L} exp( 2ipi f(n)) biggr|^2 le (LW^{-1/3} +eta) (alpha L + beta W^{2/3})` where $\alpha = \frac{1}{\eta} + \frac{64\lambda}{75}\sqrt{\eta + W^{-1/3}} + \frac{\lambda\eta}{W^{1/3}} + \frac{\lambda}{W^{2/3}}$, and $\beta = \frac{65}{15\sqrt{\eta}} + \frac{3}{W^{1/3}}$.

5.4 4. Bounds with higher derivatives

See Lemma 3.1 and 3.2 of [Patel, 2022].

5.5 5. Iterated Van der Corput Inequality

During the proof of Lemma 8.6 in
[Granville and Ramaré, 1996] one finds the next inequality.

Theorem (1996)

Let f be a real-valued function with $k+1$ continuous derivatives on $(A, B]$ and let N be a lower bound for the number of integers in $(A, B]$. The quantity $\left|\frac{1}{8N}\sum_{A<n\leq B}\exp(2i\pi f(n))\right|^{2^k}$ is bounded above by $\frac{1}{8}\Bigg(\frac{1}{Q} + \frac{1}{Q^{2-2^{1-k}}}\sum_{r_1=1}^{Q2^{-0}}\sum_{r_2=1}^{Q2^{-1}}\cdots\sum_{r_k=1}^{Q2^{-k+1}}\left|\frac{1}{N}\sum_{A<n\leq B-r_1-r_2-\cdots-r_k}\exp(\pm 2i\pi f_{r_1,\cdots,r_k}(n))\right|\Bigg)$ where the function $f_{r_1,\cdots,r_k}$ satisfies $\forall t,\ \exists y\in[t, t+r_1+\cdots+r_k],\quad f'_{r_1,\cdots,r_k}(t) = r_1 r_2\cdots r_k f^{(k+1)}(y)$.

5.6 6. Explicit Poisson Formula

Here is a consequence of the main theorem of [Karatsuba and Korolëv, 2007].

Theorem (2007)

Suppose f' is decreasing on $[N+1, N+L]$ and set $f'(N+L) = \alpha$ and $f'(N) = \beta$. For integer $\nu \in (\alpha, \beta]$, let x_ν be the solution to $f'(x) = \nu$. Suppose further that $\lambda_2 \leq |f''(x)| \leq h_2\lambda_2$ and $\lambda_3 \leq |f'''(x)| \leq h_3\lambda_3$. Then $\sum_{n=N+1}^{N+L}\exp(2i\pi f(n)) = \sum_{\alpha<\nu\leq\beta}\frac{\exp(2i\pi(f(x_\nu) - \nu x_\nu - 1/8))}{\sqrt{f''(x_\nu)}} + \mathcal{E}$ where $|\mathcal{E}| \leq \frac{40}{\sqrt{\pi}}\lambda^{-1/2} + \frac{3+2h_2}{\pi}\log(\beta-\alpha+2) + 2.9h_2h_3^{1/5}L(\lambda_2\lambda_3)^{1/5} + 1.9$.

6

Bounds for the Gamma function

6.1 1. Bounds for $\Gamma(x)$ for real x

The classical bounds (Stirling formula) that results from the Euler-Maclaurin formula for positive integers n and $m \geq 0$ are

These inequalities were described in words in more general form in [Bendersky, 1933] and subsequently rediscovered several times [Choi, 2012], [Camargo, 2024].

[Alzer, 1997] presents an extension of the inequalities above for real arguments in lighlty different form (see also remark 2.1 of [Chen, 2016]).

Theorem (1997)

For real $x > 0$ and $m \geq 0$, we have $\sum_{j=1}^{2m} \frac{B_{2j}}{2j(2j-1)x^{2j-1}} \leq \log\Gamma(x) - \left(x - \frac{1}{2}\right)\log(x) + x - \log(\sqrt{2\pi}) \leq \sum_{j=1}^{2m+1} \frac{B_{2j}}{2j(2j-1)x^{2j-1}}$.

There are also several bounds for $\Gamma(x)$ with a fixed number of terms available in the literature. For instance, in p. 118 of Ramanujan's lost notebook [Andrews and Berndt, 2013], one finds

Bounds of similar shape was obtaineb by [Mortici, 2011].

Theorem (2011)

For $\alpha = \frac{128}{1215} \approx 0.105349$, $\beta = \frac{218336}{135} - \frac{256e^{24}}{43046721\pi^4} \approx 0.087944$ and $x \geq 3$, we have $\sqrt[8]{16x^4 + \frac{32}{3}x^3 + \frac{32}{9}x^2 - \frac{176}{405}x - \alpha} \leq \frac{\Gamma(x+1)}{\sqrt{\pi}\left(\frac{x}{e}\right)^x} \leq \sqrt[8]{16x^4 + \frac{32}{3}x^3 + \frac{32}{9}x^2 - \frac{176}{405}x - \beta}$.

Improving on Ramanujan's and some other estimates, [Chen, 2016] obtained the following result.

Theorem (2016)

For $x \geq 2$, we have $1 - \frac{2117}{5080320x^7} \leq \frac{\Gamma(x+1)}{\sqrt{2\pi x}\left(\frac{x}{e}\right)^x\left(1+\frac{1}{12x^3+\frac{24}{7}x-\frac{1}{2}}\right)^{x^2+\frac{53}{210}}} \leq 1 - \frac{2117}{5080320x^7} + \frac{1892069}{2347107840x^9}$.

Meanwhile, [Batir, 2008] presented some estimates for $\Gamma(x)$ of certain shapes with optimal constants.

Theorem (2008)

For $x > 0$, $a = \sqrt{2e} = 2.33164$, and $b = \sqrt{2\pi} = 2.50662...$ we have $a\left(\frac{x+1/2}{e}\right)^{x+1/2} \leq \Gamma(x+1) < b\left(\frac{x+1/2}{e}\right)^{x+1/2}$. The constants a and b are the best possible.

Theorem (2008)

For $x \geq 1$, $a = 1/6$, and $b = \frac{e^2}{2\pi} - 1 = 0.176005...$ we have $x^x e^{-x}\sqrt{2\pi(x+a)} < \Gamma(x+1) < x^x e^{-x}\sqrt{2\pi(x+b)}$. The constants a and b are the best possible.

6.2 2. Bounds for the Digamma function $\psi(x) = \Gamma'(x)/\Gamma(x)$ for real x

Let

In [Diamond and Straub, 2016], the authors prove the following result.

Theorem (2016)

Let $N \geq 1$ and let λ_0 be the unique root of $B_N(\lambda)$ in $[0, 1/2]$ for N even or the unique root of $B_{N+1}(\lambda)$ for N odd. For $x > \lambda$, we have $\begin{cases} \psi(x) > F_N(\lambda, x), & N \equiv 1 \mod 4, \ \lambda \in [\lambda_0, 1/2]; \\ \psi(x) < F_N(\lambda, x), & N \equiv 3 \mod 4, \ \lambda \in [\lambda_0, 1/2]; \\ \psi(x) > F_N(\lambda, x), & N \equiv 2 \mod 4, \ \lambda \in [0, \lambda_0]; \\ \psi(x) < F_N(\lambda, x), & N \equiv 0 \mod 4, \ \lambda \in [0, \lambda_0]. \end{cases}$

The first two cases of the theorem above for $\lambda = 0$ yields the following result which was previously published in [Gordon, 1994].

Some variants of these inequalities for $\psi(x)$ can be found in the literature. An example is the following.

Theorem ([Mortici, 2011])

For $x \geq 1$, we have $-\frac{1}{24x^2} + \frac{1}{12x^3} - \frac{337}{2280x^4} < \psi(x+1) + \log\left(e^{1/(x+1)} - 1\right) < -\frac{1}{24x^2} + \frac{1}{12x^3} - \frac{337}{2280x^4} + \frac{97}{720x^5}$.

6.3 3. Bounds for the Polygamma functions $\psi(x)', \psi(x)'', \psi(x)'''...$ for real x

Let

[], proves the following result.

Theorem (1997)

For $x > 0, k \geq 1$ and $n \geq 0$, we have $S_k(2n, x) < (-1)^{k+1}\psi^{(k)}(x) < S_k(2n+1, x)$.

The first two cases of the theorem above are

Similar inequalities were previously obtained by [Gordon, 1994]. For $x > 0$,

Similar inequalities to those of [] were obtained in equation (3.4) of []

Theorem (2002)

For $x \geq 1/2$, we have $\frac{(n-1)!}{(x-\frac{1}{2})^n} + \sum_{k=1}^{2N+1} \frac{B_{2k}(1/2)}{(2k)!} \frac{(n+2k+1)!}{(x-\frac{1}{2})^{n+2k}} < (-1)^{(n+1)}\psi^{(n)}(x) < \sum_{k=1}^{2N} \frac{B_{2k}(1/2)}{(2k)!} \frac{(n+2k+1)!}{(x-\frac{1}{2})^{n+2k}}$.

The values $B_{2k}(1/2)$ in the theorem above are known in explicit form ([Allasia *et al.*, 2002])

Several other inequalities for $\psi^{(k)}(x)$ can be obtained exploring the relations between $\psi^{(k)}(x)$ and $\psi(x)$. For instance, in [Guo and Qi, 2013], one finds (among other things)

6.4 4. Complex Stirling formula

The complex version of the Euler-maclaurin formula for the Gamma function (see, e.g., [Lang, 1999], p. 422) is

which holds for all nonzero complex numbers s which does not have negative real part. In the relation above, log means the principal branch of the logarithm and $B_1(x) = x - \frac{1}{2}$ is the Bernoulli polynomial of degree one. Using similar expressions with more terms of the Euler-Maclaurin formula, one might be able to deduce the complex version of Stirling's formula (see [Gradshteyn and Ryzhik, n.d.], 8.344):

with

A few other estimates can be found in Section 20.2 of [Ramaré, 2021]. For $0 < \delta < \pi$ and $s = |s|e^{i\phi}$ with $|\phi| \leq \pi - \delta$, the following holds

For $s = \sigma + it$ with $\sigma > 0$, we also have

and

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A few other estimates can be found in Section 20.2 of [Ramaré, 2021]. For $0 < \delta < \pi$ and $s = |s|e^{i\phi}$ with $|\phi| \leq \pi - \delta$, the following holds

For $s = \sigma + it$ with $\sigma > 0$, we also have

and

7 Bounds for $|\zeta(s)|$, $|L(s,\chi)|$ and related questions

Collecting references: [Trudgian, 2011], [Kadiri and Ng, 2012].

7.1 1. Approximating $|\zeta(s)|$ or L-series in the critical strip

[Arias de Reyna, 2011] extends the Phd memoir [Gabcke, 1979] and provides an explicit Riemann-Siegel formula for $\zeta(s)$.

Theorem 1.2 of [Kadiri, 2013] proves the following.

Theorem (2013)

When $t > t_0 > 0$, $c > 1/(2\pi)$ and $s = \sigma + it$ with $\sigma \geq 1/2$, we have $\zeta(s) = \sum_{n<ct} \frac{1}{n^s} + \mathcal{O}^*\left(\left(c + \frac{1}{2} + \frac{3\sqrt{1+1/t_0^2}}{2\pi}\left(\frac{\pi}{12c} + 1 + \frac{1}{2\pi c - 1}\right)\right)(ct)^{-\sigma}\right)$.

Notice that, by using the constant c, we may deduce from this an approximation of $\zeta(s)$ by a fixed Dirichlet polynomial when $T \leq t \leq 2T$, for some parameter T.

7.2 2. Size of $|\zeta(s)|$ and of L-series

Theorem 4 of [Rademacher, 1959] gives the convexity bound. See also section 4.1 of [Trudgian, 2014].

Theorem (1959)

In the strip $-\eta \leq \sigma \leq 1 + \eta$, $0 < \eta \leq 1/2$, the Dedekind zeta function $\zeta_K(s)$ belonging to the algebraic number field K of degree n and discriminant d satisfies the inequality $|\zeta_K(s)| \leq 3\left|\frac{1+s}{1-s}\right|\left(\frac{|d||1+s|}{2\pi}\right)^{\frac{1+\eta-\sigma}{2}}\zeta(1+\eta)^n$.

On the line $\Re s = 1/2$, Lemma 2 of [Lehman, 1970] gives a better result, namely

Theorem (1970)

If $t \geq 1/5$, we have $|\zeta(\frac{1}{2}+it)| \leq 4(t/(2\pi))^{1/4}$.

In fact, Lehman states this Theorem for $t \geq 64/(2\pi)$, but modern means of computations makes it easy to check that it holds as soon as $t \geq 0.2$. See also equation (56) of [Backlund, 1918] reproduced below.

For Dirichlet L-series, Theorem 3 of [Rademacher, 1959] gives the corresponding convexity bound.

Theorem (1959)

In the strip $-\eta \leq \sigma \leq 1+\eta$, $0 < \eta \leq 1/2$, for all moduli $q > 1$ and all primitive characters χ modulo q, the inequality $|L(s,\chi)| \leq \left(q\frac{|1+s|}{2\pi}\right)^{\frac{1+\eta-\sigma}{2}}\zeta(1+\eta)$ holds.

This paper contains other similar convexity bounds.

Corollary to Theorem 3 of [Cheng and Graham, 2004] goes beyond convexity.

Theorem (2001)

For $0 \leq t \leq e$, we have $|\zeta(\frac{1}{2}+it)| \leq 2.657$. For $t \geq e$, we have $|\zeta(\frac{1}{2}+it)| \leq 3t^{1/6}\log t$. Section 5 of bibref("Trudgian*13") shows that one can replace the constant 3 by 2.38.

This is improved in [Hiary, 2016].

Theorem (2016)

When $t \geq 3$, we have $|\zeta(\frac{1}{2}+it)| \leq 0.63t^{1/6}\log t$.

Concerning L-series, the situation is more difficult but [Hiary, 2016] manages, among other and more precise results, to prove the following.

Theorem (2016)

Assume χ is a primitive Dirichlet character modulo $q > 1$. Assume further that q is a sixth power. Then, when $|t| \geq 200$, we have $|L(\frac{1}{2}+it,\chi)| \leq 9.05d(q)(q|t|)^{1/6}(\log q|t|)^{3/2}$ where $d(q)$ is the number of divisors of q.

It is often useful to have a representation of the Riemann zeta function or of L-series inside the critical strip. In the case of L-series, [Spira, 1969] and [Rumely, 1993] proceed via decomposition in Hurwitz zeta function which they compute through an Euler-MacLaurin development. We have a more efficient approximation of the Riemann zeta function provided by the Riemann Siegel formula, see for instance equations (3-2)–(3.3) of [Odlyzko, 1987]. This expression is due to [Gabcke, 1979]. See also equations (2.4)-(2.5) of [Lehman, 1966], a corrected version of Theorem 2 of [Titchmarsh, 1947].

In general, we have the following estimate taken from equations (53)-(54), (56) and (76) of [Backlund, 1918] (see also [Backlund, 1914]).

Theorem (1918)

- When $t \geq 50$ and $\sigma \geq 1$, we have $|\zeta(\sigma+it)| \leq \log t - 0.048$.
- When $t \geq 50$ and $0 \leq \sigma \leq 1$, we have $|\zeta(\sigma+it)| \leq \frac{t^2}{t^2-4}\left(\frac{t}{2\pi}\right)^{\frac{1-\sigma}{2}} \log t$.
- When $t \geq 50$ and $-1/2 \leq \sigma \leq 0$, we have $|\zeta(\sigma+it)| \leq \left(\frac{t}{2\pi}\right)^{\frac{1}{2}-\sigma} \log t$.

On the line $\Re s = 1$, [Trudgian, 2014] establishes the next result.

Theorem (2012)

When $t \geq 3$, we have $|\zeta(1+it)| \leq \frac{3}{4}\log t$.

The paper [Patel, 2022] proves the next bound.

Theorem (2022)

When $t \geq 3$, we have $|\zeta(1+it)| \leq \min\left(\frac{3}{4}\log t, \frac{1}{2}\log t + 1.93, \frac{1}{5}\log t + 44.02\right)$.

Asymptotically better bounds are available since the work of [Ford, 2002].

Theorem (2002)

When $t \geq 3$ and $1/2 \leq \sigma \leq 1$, we have $|\zeta(\sigma+it)| \leq 76.2 t^{4.45(1-\sigma)^{3/2}}(\log t)^{2/3}$.

The constants are still too large for this result to be of use in any decent region. See [Kulas, 1994] for an earlier estimate.

7.3 3. On the total number of zeroes

The first explicit estimate for the number of zeros of the Riemann ζ-function goes back to [Backlund, 1914]. An elegant consequence of the result of Backlund is the following easy estimate taken from Lemma 1 of [Lehman, 1966].

Theorem (1966)

If φ is a continuous function which is positive and monotone decreasing for $2\pi e \leq T_1 \leq t \leq T_2$, then $\sum_{T_1<\gamma\leq T_2} \varphi(\gamma) = \frac{1}{2\pi}\int_{T_1}^{T_2} \varphi(t)\log\frac{t}{2\pi}dt + O^*\left(4\varphi(T_1)\log T_1 + 2\int_{T_1}^{T_2}\frac{\varphi(t)}{t}dt\right)$ where the summation is over all zeros of the Riemann ζ-function of imaginary part between T_1 and T_2, with multiplicity.

Theorem 19 of [Rosser, 1941] gives a bound for the total number of zeroes.

Theorem (1941)

For $T \geq 2$, we have $N(T) = \sum_{\substack{\rho, \\ 0<\gamma\leq T}} 1 = \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi} + \frac{7}{8} + O^*\Big(0.137\log T + 0.443\log\log T + 1.588\Big)$ where the summation is over all zeros of the Riemann ζ-function of imaginary part between 0 and T, with multiplicity.

It is noted in Lemma 1 of [Ramaré and Saouter, 2003] that the O-term can be replaced by the simpler $O^*(0.67\log\frac{T}{2\pi})$ when $T \geq 10^3$.

This is improved in Corollary 1 of [Trudgian, 2014] into

Theorem (2014)

For $T \geq e$, we have $N(T) = \sum_{\substack{\rho, \\ 0<\gamma\leq T}} 1 = \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi} + \frac{7}{8} + O^*\big(0.112\log T + 0.278\log\log T + 2.510 + \frac{1}{5T}\big)$ where the summation is over all zeros of the Riemann ζ-function of imaginary part between 0 and T, with multiplicity.

Corollary 1.4 of the main theorem of [Hasanalizade *et al.*, 2022] reads

Theorem (2022)

For $T \geq e$, we have $N(T) = \sum_{\substack{\rho, \\ 0<\gamma\leq T}} 1 = \frac{T}{2\pi}\log\frac{T}{2\pi} - \frac{T}{2\pi} + \frac{7}{8} + O^*\big(0.1038\log T + 0.2573\log\log T + 9.3675\big)$ where the summation is over all zeros of the Riemann ζ-function of imaginary part between 0 and T, with multiplicity. We may also replace $0.1038\log T + 0.2573\log\log T + 9.3675$ by $0.1095\log T + 0.2042\log\log T + 3.0305$.

Concerning Dirichlet L-functions, the paper [Bennett *et al.*, 2021] contains the next result.

Theorem (2021)

Let χ be a Dirichlet character of conductor $q > 1$. For $T \geq 5/7$ and $\ell = \log\frac{q(T+2)}{2\pi} > 1.567$, we have $N(T,\chi) = \sum_{\substack{\rho, \\ 0<\gamma\leq T}} 1 = \frac{T}{\pi}\log\frac{qT}{2\pi} - \frac{T}{\pi} + \frac{\chi(-1)}{4} + O^*\big(0.22737\ell + 2\log(1+\ell) - 0.5\big)$ where the summation is over all zeros of the Dirichlet function $L(\cdot,\chi)$ of imaginary part between $-T$ and T, with multiplicity.

7.4 4. L^2-averages

In Theorem 1.4 of [Kadiri, 2013], we find the next result.

Theorem (2013)

When $0.5208 < \sigma < 0.9723$ and $10^3 \le H \le 10^{10}$, we have, for any $T \ge H$, $\int_H^T |\zeta(\sigma+t)|^2 dt \le (T-H)\big(\zeta(2\sigma)+\mathcal{E}_1(\sigma,H)\big)$ where $\mathcal{E}_1(\sigma,H)$ is a small error term whose precise expression in given the stated paper.

We can find in [] the proof of the following estimate. Though it is unpublished yet, the full proof is available.

Theorem (2019)

Let $0 < \sigma \le 1$ and $T \ge 3$. Then $\frac{1}{2\pi}\left(\int_{\sigma-i\infty}^{\sigma-iT} + \int_{\sigma+iT}^{\sigma+i\infty}\right)\frac{|\zeta(s)|^2}{|s|^2}ds \le \kappa_{\sigma,T}\begin{cases}\frac{c_{1,\sigma}}{T}+\frac{c_{1,\sigma}^\flat}{T^{2\sigma}} & \text{when } \sigma > 1/2,\\ \frac{\log T}{2T}+\frac{c_{2,\sigma}^\flat}{T} & \text{when } \sigma = 1/2,\\ c_{3,\sigma}/T^{2\sigma} & \text{when } \sigma < 1/2.\end{cases}$ where $c_{1,\sigma} = \zeta(2\sigma)/2, c_{1,\sigma}^\flat = c^2\frac{3^{2\sigma}}{2\sigma}, c_{2,\sigma}^\flat = 3c^2 + \frac{1-\log 3}{2}, c = 9/16$ $c_{3,\sigma} = \left(\frac{c^2}{2\sigma}+\frac{1/6}{1-2\sigma}\right)\left(1+\frac{1}{\sigma}\right)^{2\sigma}, \kappa_{\sigma,T} = \begin{cases}\frac{9/4}{\left(1-\frac{9/2}{T^2}\right)^2} & \text{when } 1/2 \le \sigma \le 1,\\ \frac{(1+\sigma)^2}{\left(1-\frac{(1+\sigma)^2}{\sigma T^2}\right)^2} & \text{when } 0 < \sigma < 1/2.\end{cases}$

7.5 5. Bounds on the real line

After some estimates from [Bastien and Rogalski, 2002] Lemma 5.1 of [Ramaré, 2016] shows the following.

Theorem (2013)

When $\sigma > 1$ and t is any real number, we have $|\zeta(\sigma+it)| \le e^{\gamma(\sigma-1)}/(\sigma-1)$.

Here is the Theorem of [Delange, 1987]. See also Lemma 2.3 of [Ford, 2000] for a slightly weaker version.

Theorem (1987)

When $\sigma > 1$ and t is any real number, we have $-\Re\frac{\zeta'}{\zeta}(\sigma+it) \le \frac{1}{\sigma-1} - \frac{1}{2\sigma^2}$.

8

Size of $L(1, \chi)$

Collecting references: [Louboutin, 1993],

8.1 1. Upper bounds for $|L(1, \chi)|$

[Louboutin, 1996], [Louboutin, 1998], [Granville and Soundararajan, 2003], [Granville and Soundararajan, 2004]. [Ramaré, 2001],

Theorem (2001)

For any primitive Dirichlet character χ of conductor q, we have $|L(1,\chi)| \leq \frac{1}{2}\log q + \begin{cases} 0 & \text{when } \chi \text{ is even (i.e. } \chi(-1)=1),\\ \frac{5}{2}-\log 6 & \text{when } \chi \text{ is odd (i.e. } \chi(-1)=-1). \end{cases}$

When the conductor q is even, this may be improved.

Theorem (2001)

For any primitive Dirichlet character χ of even conductor q, we have $|L(1,\chi)| \leq \frac{1}{4}\log q + \begin{cases} \frac{1}{2}\log 2 & \text{when } \chi \text{ is even (i.e. } \chi(-1)=1),\\ \frac{5}{4}-\frac{1}{2}\log 3 & \text{when } \chi \text{ is odd (i.e. } \chi(-1)=-1). \end{cases}$

Similar bounds or more precise bounds may be found in [Louboutin, 2002], in [Ramaré, 2004] and in [Louboutin, 2018],

In [Platt and Eddin, 2013], we find the next result.

Theorem (2013)

For any primitive Dirichlet character χ of conductor q divisible by 3, we have $|L(1,\chi)| \leq \frac{1}{3}\log q + \begin{cases} 0.368296 & \text{when } \chi \text{ is even (i.e. } \chi(-1)=1),\\ 0.838374 & \text{when } \chi \text{ is odd (i.e. } \chi(-1)=-1).\end{cases}$

In [Saad Eddin, 2016], we find improvement on the $(1/2)\log q$ bound in a very special (and difficult) case.

Theorem (2016)

For any primitive Dirichlet even character χ of conductor q and with $\chi(2) = 1$, we have $|L(1,\chi)| \leq \frac{1}{2}\log q - 0.02012$.

The general upper bounds are improved in [Johnston *et al.*, 2023] as follows.

Theorem (2023)

For any quadratic primitive Dirichlet character χ of conductor $f \geq 2\cdot 10^{23}$, we have $|L(1,\chi)| \leq (1/2)\log f$.

Theorem (2023)

For any quadratic primitive Dirichlet character χ of conductor $f \geq 5\cdot 10^{50}$, we have $|L(1,\chi)| \leq (9/20)\log f$. When f is even, the lower bound on f may be improved to $f \geq 2\cdot 10^{49}$.

8.2 2. Lower bounds for $|L(1,\chi)|$

[Louboutin, 2013] announces the following lower bound proved in [Louboutin, 2015].

Theorem (2013)

For any non-quadratic primitive Dirichlet character χ of conductor f, we have $|L(1,\chi)| \geq 1/(10\log(f/\pi))$.

This is improved in [Mossinghoff *et al.*, 2022] where we find the next bound.

Theorem (2022)

For any non-quadratic primitive Dirichlet character χ of conductor f, we have $|L(1,\chi)| \geq 1/(9.69030\log(f/\pi))$.

9

Explicit zero-free regions for the ζ and L functions

9.1 1. Numerical verifications of the Generalized Riemann Hypothesis

Numerical verifications of the Riemann hypothesis for the Riemann ζ-function have been pushed extremely far. B. Riemann himself computed the first zeros. Concerning more recent published papers, we mention the next result.

Theorem ([Van de Lune *et al.*, 1986])

Every zero ρ of ζ that have a real part between 0 and 1 and an imaginary part not more, in absolute value, than $\leq T_0 = 545\,439\,823$ are in fact on the critical line, i.e. satisfy $\Re\rho = 1/2$.

The bound $545\,439\,823$ is increased to $1\,000\,000\,000$ in [Platt, 2011]. In [Platt, 2017], this bound is further increased to $30\,610\,046\,000$. Between these results, a group using a network method announced:

Theorem ([Wedeniwski, 2002])

$T_0 = 29\,538\,618\,432$ is admissible in the theorem above.

Here is another announcement.

Theorem ([Gourdon and Demichel, 2004])

$T_0 = 2.445 \cdot 10^{12}$ is admissible in the theorem above.

These two last announcements have not been subject to any academic papers.

Theorem ([Platt and Trudgian, 2021])

$T_0 = 3 \cdot 10^{12}$ is admissible in the theorem above.

One of the key ingredient is an explicit Riemann-Siegel formula due to [Gabcke, 1979] (the preprint of Gourdon mentionned above gives a version of Gabcke's result) and such a formula is missing in the case of Dirichlet L-function.

Let us introduce some terminology. We say that a modulus $q \geq 1$ (i.e. an integer!) satisfies $GRH(H)$ for some numerical value H when every zero ρ of the L-function associated to a primitive Dirichlet character of conductor q and whose real part lies within the critical line (i.e. has a real part lying inside the open interval $(0,1)$) and whose imaginary part is below, in absolute value, H, in fact satisfies $\Re\rho = 1/2$.

The next result was proved by employing an Euler-McLaurin formula.

Theorem ([Rumely, 1993])

- Every $q \leq 13$ satisfies $GRH(10\,000)$.
- Every q belonging to one of the sets
- $\{k \leq 72\}$
- $\{k \leq 112 : k \text{ not prime}\}$
- $\{116, 117, 120, 121, 124, 125, 128, 132, 140, 143, 144, 156, 163, 169, 180, 216, 243, 256, 360, 420, 432\}$

satisfies $GRH(2\,500)$.

These computations have been extended by [Bennett, 2001] by using Rumely's programm. All these computations have been superseded by the work of D. Platt. who uses two fast Fourier transforms, one in the t-aspect and one in the q-aspect, as well as an approximate functionnal equation to prove via extremely rigorous computations that

Theorem ([Platt, 2011], [Platt, 2013])

Every modulus $q \leq 400\,000$ satisfies $GRH(100\,000\,000/q)$.

We mention here the algorithm of [Omar, 2001] that enables one to prove efficiently that some L-functions have no zero within the rectangle $1/2 \leq \sigma \leq 1$ et $2\sigma - |t| = 1$ though this algorithm has not been put in practice.

There are much better results concerning real zeros of Dirichlet L-functions associated to real characters.

9.2 2. Asymptotical zero-free regions

The first fully explicit zero free region for the Riemann zeta-function is due to [Rosser, 1938] in Lemma 19 (essentially with $R_0 = 19$ in the notations below). This is next imporved upon in Theorem 1 of [Rosser and Schoenfeld, 1975] by using a device of [Stechkin, 1970] (getting essentially $R_0 = 9.646$). The next step is in [Ramaré and Rumely, 1996] where the second order term is improved upon, relying on [Stechkin, 1989].

Next, the following result is proven.

Theorem ([Kadiri, 2002], [Kadiri, 2005])

The Riemann ζ-function has no zeros in the region $\Re s \geq 1 - \dfrac{1}{R_0 \log(|\Im s| + 2)}$ with $R_0 = 5.70175$.

[Jang and Kwon, 2014] improved the value of R_0 by showing that $R_0 = 5.68371$ is admissible. By plugging a better trigonometric polynomial in the same method, it is proved in [Mossinghoff and Trudgian, 2015] that

Theorem (2015)

The Riemann ζ-function has no zeros in the region $\Re s \geq 1 - \dfrac{1}{R_0 \log(|\Im s| + 2)}$ with $R_0 = 5.573412$.

Concerning Dirichlet L-function, the first explicit zero-free region has been obtained in [McCurley, 1984] by adaptating [Rosser and Schoenfeld, 1975]. [Kadiri, 2002] (cf also [Kadiri, 2009]) improves that into:

Theorem (2002)

The Dirichlet L-functions associated to a character of conductor q has no zero in the region: $\Re s \geq 1 - \dfrac{1}{R_1 \log(q \max(1, |\Im s|))}$ with $R_1 = 6.4355$, to the exception of at most one of them which would hence be attached to a real-valued character. This exceptional one would have at most one zero inside the forbidden region (and which is loosely called a "Siegel zero").

Theorem ([Kadiri, 2018])

The Dirichlet L-functions associated to a character of conductor $q \in [3, 400\,000]$ has no zero in the region: $\Re s \geq 1 - \dfrac{1}{R_2 \log(q \max(1, |\Im s|))}$ with $R_2 = 5.60$.

Here is an explicit Vinogradov-Korobov zero-free region.

Theorem ([Ford, 2000])

The Riemann ζ-function has no zeros in the region $\Re s \geq 1 - \dfrac{1}{58(\log |\Im s|)^{2/3}(\log\log |\Im s|)^{1/3}}$ $(|\Im s| \geq 3)$.

A Vinogradov-Korobov zero-free region for Dirchlet L-functions has later been obtained.

Theorem ([Khale, 2024])

The Riemann ζ-function has no zeros in the region $\Re s \geq 1 - \dfrac{1}{10.5\log q + 61.5(\log|\Im s|)^{2/3}(\log\log|\Im s|)^{1/3}}$ $(|\Im s| \geq 10)$.

If we are ready to assume $|\Im s|$ large enough, the same proof can reduce the 61.5 to 49.13.

Concerning the Dedekind ζ-function, see [Kadiri, 2012].

9.3 3. Real zeros

[Rosser, 1949], [Rosser, 1950], [Chua, 2005], [Watkins, 2004],

Theorem ([Ralaivaosaona and Razakarinoro, 2026])

Let $d > 3\cdot 10^8$ such that $-d$ is a fundamental discriminant. The Dirichlet series $L(s,\chi_d)$ associated to the primitive character :math:*chi_d(n)=(frac{-d}{n})* has not zero in the half-plane $\Re s \geq 1 - 6.035/\sqrt{d}$.

9.4 4. Density estimates

After initial work of [Chen and Wang, 1989] and [Liu and Wang, 2002], here are the latest two best results. We first define

$N(\sigma,T,\chi) = \sum_{\substack{\rho=\beta+i\gamma,\\ L(\rho,\chi)=0,\\ \sigma\leq\beta,|\gamma|\leq T}} 1$ which thus counts the number of zeroes ρ of $L(s,\chi)$, zeroes whose real part is denoted by β (and assumed to be larger than σ), and whose imaginary part in absolute value γ is assumed to be not more than T. For the Riemann ζ-function (i.e. when $\chi = \chi_0$ the principal character modulo~1), it is customary to count only the zeroes with positive imaginary part. The relevant number is usually denoted by $N(\sigma,T)$. We have $2N(\sigma,T) = N(\sigma,T,\chi_0)$.

For low values, we start with the next theorem. We reproduce only a special case.

Theorem ([Kadiri and Ng, 2012])

Let $T \geq 3.061\cdot 10^{10}$. We have $2N(17/20,T,\chi_0) \leq 0.5561T + 0.7586\log T - 268658$ where χ_0 is the principal character modulo 1.

See also [Kadiri, 2013]. Otherwise, here is the result of [Ramaré, 2016].

Theorem (2016)

For $T \geq 2\,000$ and $T \geq Q \geq 10$, as well as $\sigma \geq 0.52$, we have $\sum_{q\leq Q}\frac{q}{\varphi(q)}\sum_{\chi \bmod {}^*q} N(\sigma,T,\chi) \leq 20\big(56\,Q^5T^3\big)^{1-\sigma}\log^{5-2\sigma}(Q^2T) + 32\,Q^2\log^2(Q^2T)$ where $\chi \bmod {}^*q$ denotes a sum over all primitive Dirichlet character χ to the modulus q. Furthermore, we have $N(\sigma,T,\chi_0) \leq$

$6T\log T\log\left(1+\frac{6.87}{2T}(3T)^{8(1-\sigma)/3}\log^{4-2\sigma}(T)\right)+103(\log T)^2$ where χ_0 is the principal character modulo 1.

In [Kadiri *et al.*, 2018]. this result is improved upon, we refer to their paper for their result but quote a corollary.

Theorem (2018)

For $T\geq 1$, we have $N(0.9,T)\leq 1.5\,T^{4/14}\log^{16/5}(T)+3.2\,\log^2 T$ where $N(\sigma,T)=N(\sigma,T,\chi_0)$ and χ_0 is the principal character modulo 1.

9.5 5. Miscellanae

The LMFDBfootnote{url{http://www.lmfdb.org}} database contains the first zeros of many L-functions. A part of Andrew Odlyzko's

websitefootnote{url{http://www.dtc.umn.edu/~odlyzko/zeta_tables/index.html#GlobalIndex}} contains extensive tables concerning zeroes of the Riemann zeta function.

10

Short intervals containing primes

10.1 1. Interval with primes, without congruence condition, saving a constant

The story seems to start in 1845 when Bertrand conjectured after numerical trials that the interval $(n, 2n-3]$ contains a prime as soon as $n \geq 4$. This was proved by Pafnouti Tchebychev in 1852 in a famous work where he got the first good quantitative estimates for the number of primes less than a given bound, say x. By now, analytical means combined with sieve methods (see [Baker *et al.*, 2001]) ensures us that each of the intervals $[x, x+x^{0.525}]$ for $x \geq x_0$ contains at least one prime. This statement concerns only for the (very) large integers.

It falls very close to what we can get under the assumption of the Riemann Hypothesis: the interval $[x - K\sqrt{x}\log x, x]$ contains a prime, where K is an effective large constant and x is sufficiently large (cf [Wolke, 1983] for an account on this subject). A theorem of Schoenfeld [Schoenfeld, 1976] also tells us that the interval

$[x-\sqrt{x}\log^2 x/(4\pi), x]$ contains a prime for $x \geq 599$ under the Riemann Hypothesis. These results are still far from the conjecture in [Cramer, 1936] on probabilistic grounds: the interval $[x - K\log^2 x, x]$ contains a prime for any $K > 1$ and $x \geq x_0(K)$. Note that this statement has been proved for almost all intervals in a quadratic average sense in [Selberg, 1943] assuming the Riemann Hypothesis and replacing K by a function $K(x)$ tending arbitrarily slowly to infinity.

[Schoenfeld, 1976] proved the following.

Theorem (1976)

Let x be a real number larger than $2\,010\,760$. Then the interval $\left(x, x\left(1+\frac{1}{16\,597}\right)\right)$ contains at least one prime.

The main ingredient is the explicit formula and a numerical verification of the Riemann hypothesis.

From a numerical point of view, the Riemann Hypothesis is known to hold up to a very large height (and larger than in 1976). [Wedeniwski, 2002] and the Zeta grid project verified this hypothesis till height $T_0 = 2.41 \cdot 10^{11}$ and [Gourdon and Demichel, 2004] till height $T_0 = 2.44 \cdot 10^{12}$ thus extending the work [Van de Lune *et al.*, 1986] who had conducted such a verification in 1986 till height 5.45×10^8. This latter computations has appeared in a refereed journal, but this is not the case so far concerning the other computations; section 4 of the paper [Saouter and Demichel, 2010] casts some doubts on whether all the zeros where checked. Discussions in 2012 with Dave Platt from the university of Bristol led me to believe that the results of [Wedeniwski, 2002] can be replicated in a very rigorous setting, but that it may be difficult to do so with the results of [Gourdon and Demichel, 2004] with the hardware at our disposal.

In [Ramaré and Saouter, 2003], we used the value $T_0 = 3.3 \cdot 10^9$ and obtained the following.

Theorem (2002)

Let x be a real number larger than $10\,726\,905\,041$. Then the interval $\left(x\left(1-\frac{1}{28\,314\,000}\right), x\right]$ contains at least one prime.

If one is interested in somewhat larger value, the paper [Ramaré and Saouter, 2003] also contains the following.

Theorem (2002)

Let x be a real number larger than $\exp(53)$. Then the interval $\left(x\left(1-\frac{1}{204\,879\,661}\right), x\right]$ contains at least one prime.

Increasing the lower bound in x only improves the constant by less than 5 percent. If we rely on [Gourdon and Demichel, 2004], we can prove that

Theorem (2004, conditional)

Let x be a real number larger than $\exp(60)$. Then the interval $\left(x\left(1-\frac{1}{14\,500\,755\,538}\right), x\right]$ contains at least one prime.

Note that all prime gaps have been computed up to 10^{15} in [Nicely, 1999], extending a result of [Young and Potler, 1989].

The above theorem is improved in ` <Biblio.html#>`_. We find in particular the next result.

Theorem (2014)

Let x be a real number larger than $\exp(60)$. Then the interval $\left(x\left(1-\frac{1}{1\,966\,196\,911}\right), x\right]$ contains at least one prime.

This is further improved in ` <Biblio.html#>`_. We find in particular the next result.

Theorem (2024)

Let x be a real number larger than $\exp(60)$. Then the interval $\left(x\left(1-\frac{1}{76\,900\,000\,000}\right), x\right]$ contains at least one prime.

10.2 2. Interval with primes, no congruence condition, saving more than a constant

In [Trudgian, 2016], we find

Theorem (2016)

Let x be a real number larger than $2\,898\,242$. The interval $\left[x, x\left(1+\frac{1}{111(\log x)^2}\right)\right]$ contains at least one prime.

In [Dusart, 2018], we find

Theorem (2016)

Let x be a real number larger than $468\,991\,632$. The interval $\left[x, x\left(1+\frac{1}{5000(\log x)^2}\right)\right]$ contains at least one prime.

Let x be a real number larger than $89\,693$. The interval $\left[x, x\left(1+\frac{1}{\log^3 x}\right)\right]$ contains at least one prime.

The proof of these latter results has an asymptotical part, for $x \geq 10^{20}$ where we used the numerical verification of the Riemann hypothesis together with two other arguments: a (very strong) smoothing argument and a use of the Brun-Titchmarsh inequality.

The second part is of algorithmic nature and covers the range $10^{10} \leq x \leq 10^{20}$ and uses prime generation techniques [Maurer, 1995]: we only look at families of numbers whose primality can be established with one or two Fermat-like or Pocklington's congruences. This kind of technique has been already used in a quite similar problem in [Deshouillers *et al.*, 1998]. The generation technique we use relies on a theorem proven in [Brillhart *et al.*, 1975] and enables us to generate dense enough families for the upper part of the range to be investigated. For the remaining (smaller) range, we use theorems of [Jaeschke, 1993] that yield a fast primality test (for this limited range).

Let us recall here that a second line of approach following the original work of v Cebyv sev is still under examination though it does not give results as good as analytical means (see [Costa Pereira, 1989] for the latest result).

For very large numbers [Dudek, 2016] proved the following.

Theorem (2014)

The interval $(x, x+3x^{2/3}]$ contains a prime for $x \geq \exp(\exp(34.32))$.

This is improved in [Cully-Hugill, 2021] as follows.

Theorem (2021)

The interval $(x, x+3x^{2/3}]$ contains a prime for $x \geq \exp(\exp(33.99))$.

10.3 3. Interval with primes under RH, without any congruence condition

Theorem (2002)

Under the Riemann Hypothesis, the interval $]x - \frac{8}{5}\sqrt{x}\log x, x]$ contains a prime for $x \geq 2$.

This is improved upon in [Dudek, 2015] into:

Theorem (2015)

Under the Riemann Hypothesis, the interval $]x - \frac{4}{\pi}\sqrt{x}\log x, x]$ contains a prime for $x \geq 2$.

In [Carneiro *et al.*, 2019], the authors go one step further and prove the next result.

Theorem (2019)

Under the Riemann Hypothesis, the interval $]x - \frac{22}{25}\sqrt{x}\log x, x]$ contains a prime for $x \geq 4$.

10.4 4. Interval with primes, with congruence condition

Collecting references: [McCurley, 1984], [McCurley, 1984], [Kadiri, 2008].

11

Averages of non-negative multiplicative functions

11.1 1. Asymptotic estimates

When looking for averages of functions that look like 1 or like the divisor function, Lemma 3.2 of [Ramaré, 1995] offers an efficient easy path. The technique of comparison of two arithmetical function via their Dirichlet series is known as the Convolution method and is for instance decribed at length in [Berment and Ramaré, 2012], and in the course that can be found

herefootnote{url{https://ramare-olivier.github.io/CoursNouakchott/index.html}}.

Theorem (1995)

Let $(g_n)_{n\ge1}$, $(h_n)_{n\ge1}$ and $(k_n)_{n\ge1}$ be three complex sequences. Let $H(s) = \sum_n h_n n^{-s}$, and $\overline{H}(s) = \sum_n |h_n| n^{-s}$. We assume that $g = h \star k$, that $\overline{H}(s)$ is convergent for $\Re(s) \ge -1/3$ and further that there exist four constants A, B, C and D such that

$$\sum_{n\le t} k_n = A\log^2 t + B\log t + C + \mathcal{O}^*(Dt^{-1/3}) \text{ for } t > 0.$$

Then we have for all $t > 0$:

$$\sum_{n\le t} g_n = u\log^2 t + v\log t + w + \mathcal{O}^*(Dt^{-1/3}\overline{H}(-1/3))$$

with $u = AH(0)$, $v = 2AH'(0) + BH(0)$ and $w = AH''(0) + BH'(0) + CH(0)$. We have also

$$\sum_{n\le t} ng_n = Ut\log t + Vt + W + \mathcal{O}^*(2.5Dt^{2/3}\overline{H}(-1/3))$$

with

$$\begin{aligned} U =& 2AH(0), V = -2AH(0) + 2AH'(0) + BH(0),\\ W =& A(H''(0) - 2H'(0) + 2H(0)) + B(H'(0) - H(0)) + CH(0). \end{aligned}$$

This Lemma says that one derives information from g_n from informations on the model k_n. When this model is $k_n = 1$, the values concerning A, B and C are given by the first half of

Lemma 3.3 of [Ramaré, 1995]:

Lemma (1995)

$\sum_{n\le t} 1/n = \log t + \gamma + \mathcal{O}^*(0.9105 t^{-1/3})$.

When this model is $k_n = \tau(n)$, the number of divisors of n, the values concerning A, B and C are given by Corollary 2.2 of [Berkane *et al.*, 2012]. Please note the "$\gamma^2 - 2\gamma_1$" which is wrongly typed as "$\gamma^2 - \gamma_1$" in the aforementioned paper (and thanks to Tim Trudgian and David Platt for spotting this typo):

Lemma (2011)

$$\sum_{n\le t} \tau(n)/n = \tfrac{1}{2}\log^2 t + 2\gamma \log t + \gamma^2 - 2\gamma_1 + \mathcal{O}^*(1.16/t^{1/3})$$

where γ_1 is the second Laurent-Stieljes constant, see for instance [Kreminski, 2003] and [Coffey, 2006]. In particular, we have $\gamma_1 = -0.0728158454836767248605863758749013191377 + \mathcal{O}^*(10^{-40})$.

The constants $H(0)$, $H'(0)$ and $H''(0)$ are to be computed. In most cases, the Dirichlet series has an Euler product, in which case, (see section 3 of [Ramaré, 1995]) we have

$H(0) = \prod_p (1 + \sum_m h_{p^m})$, then

$$\frac{H'(0)}{H(0)} = \sum_p \frac{\sum_m m h_{p^m}}{1 + \sum_m h_{p^m}}(-\log p),$$

and also

$$\frac{H''(0)}{H(0)} = \left(\frac{H'(0)}{H(0)}\right)^2 + \sum_p \left\{ \frac{\sum_m m^2 h_{p^m}}{1 + \sum_m h_{p^m}} - \left[\frac{\sum_m m h_{p^m}}{1 + \sum_m h_{p^m}}\right]^2 \right\} \log^2 p.$$

It is sometimes more expedient to use the same convolution method but by comparing the function to the function $q \mapsto q$. In such a case, the next lemma, Lemma 4.3 from [Ramaré, 2016], is handy.

Theorem (2015)

We have, for any real number $x \ge 0$ and any real number $c \in [1, 2]$,

$$\sum_{q\le x} q = \tfrac{1}{2}x^2 + O^*(x^c/2).$$

This leads to the next theorem.

Theorem

Let $(h_n)_{n\geq 1}$ be a complex sequences. Let $H(s) = \sum_n h_n n^{-s}$, and $\overline{H}(s) = \sum_n |h_n| n^{-s}$. We assume that $\overline{H}(s)$ is convergent for $\Re(s) \geq c$, for some $c \in [1,2]$. Then we have for all $t > 0$:

$$\sum_{n\leq t}\sum_{d|n}\frac{n}{d}h(d) = \frac{t^2}{2}H(2) + O^*(t^c\overline{H}(c)/2).$$

A typical usage is to evaluate $\sum_{n\leq t}\phi(n)$, with $h(d) = \mu(d)$.

The convolution method has been brought one step further in [P. and Ramaré, 2017] where the following theorem is proved.

Theorem (2017)

Let $(g(m))_{m\geq 1}$ be a sequence of complex numbers such that both series $\sum_{m\geq 1} g(m)/m$ and $\sum_{m\geq 1} g(m)(\log m)/m$ converge. We define $G^\sharp(x) = \sum_{m>x} g(m)/m$ and assume that $\int_1^\infty |G^\sharp(t)|dt/t$ converges.

Let $A_0 \geq 1$ be a real parameter.

We have

$$\sum_{n\leq D}\frac{(g\star 1\!\!1)(n)}{n} = \sum_{m\geq 1}\frac{g(m)}{m}\Big(\log\frac{D}{m}+\gamma\Big) + \int_{D/A_0}^\infty G^\sharp(t)\frac{dt}{t} + O^*(\mathfrak{R})$$

where $\mathfrak{R}$ is defined by

$$\mathfrak{R} = \left|\sum_{1\leq a\leq A_0}\frac{1}{a}G^\sharp\left(\frac{D}{a}\right) + G^\sharp\left(\frac{D}{A_0}\right)\left(\log\frac{A_0}{[A_0]} - R([A_0])\right)\right| + \frac{6/11}{D}\sum_{m\leq D/A_0}|g(m)|$$

and where $[A_0]$ is the integer part of A_0, while the remainder R is defined by $R(X) = \sum_{n\leq X} 1/n - \log X - \gamma$.

The remainder $R(X)$ is shown in Lemma to verify $|R(X)| \leq \gamma/X$ for every $X > 0$, and $|R(X)| \leq (6/11)/X$ when $X \geq 1$.

Theorem 21.1 of [Ramaré, 2009] offers a fully explicit estimate for the average of a general non-negative multiplicative function, but it is often numerically rather poor. It relies on the technique developped by [Levin and Fainleib, 1967].

Theorem (2009)

Let g be a non-negative multiplicative function. Let A and κ be three positive real parameters such that, for any $Q \geq 1$, one has

$$\sum_{\substack{p\geq 2,\nu\geq 1\\ p^\nu\leq Q}} g(p^\nu)\log(p^\nu) = \kappa\log Q + \mathcal{O}^*(L)$$

and $\sum_{p\geq 2}\sum_{\nu,k\geq 1} g(p^k)g(p^\nu)\log(p^\nu) \leq A$. Then, when $D \geq \exp(2(L+A))$, we have $\sum_{d\leq D} g(d) = C(\log D)^\kappa\left(1+\mathcal{O}^*(B/\log D)\right)$ where $B = 2(L+A)\big(1+2(\kappa+1)e^{\kappa+1}\big)$ and

11.2 2. Upper bounds

When looking for an upper bound, it is common to compare sums to an Euler product, via, $\sum_{n\le y} f(n)/n \le \prod_{p\le y}\left(1+\sum_{1\le m\le \log y/\log p} f(p^m)\right)$ valid when f is non-negative and multiplicative. Lemma 4 of [Daboussi and Rivat, 2001] extends this. Let z be a parameter and $v_z(n)$ be the characteristic function of those integers that have all their prime factors $p\le z$.

Theorem (2000)

Let $z\ge 2$, f a multiplicative function with $f\ge 0$ and $S=\sum_{p\le z}\frac{f(p)}{1+f(p)}\log p$. We assume that $S>0$ and write $K(t)=\log t-1-(1/t)$. For any y such that $\log y\ge S$, we have $\sum_{n>y} v_z(n)\mu^2(n)f(n) \le \prod_{p\le z}(1+f(p))\exp\left(-\frac{\log y}{\log z}K\left(\frac{\log y}{S}\right)\right)$ $\sum_{n\le y} v_z(n)\mu^2(n)f(n) \ge \prod_{p\le z}(1+f(p))\left\{1-\exp\left(-\frac{\log y}{\log z}K\left(\frac{\log y}{S}\right)\right)\right\}$ and in particular, when $\log y\ge 7S$, we have $\sum_{n>y} v_z(n)\mu^2(n)f(n) \le \prod_{p\le z}(1+f(p))\exp\left(-\frac{\log y}{\log z}\right)$ $\sum_{n\le y} v_z(n)\mu^2(n)f(n) \ge \prod_{p\le z}(1+f(p))\left\{1-\exp\left(-\frac{\log y}{\log z}\right)\right\}$.

It is sometimes required to compare a function close to 1 (or more generally to the divisor function τ_k) to a function close to $1/n$ or $\tau_k(n)/n$. Theorem 01 of [Hall and Tenenbaum, 1988] offers a fast way of doing so.

Theorem (1988)

Let f be a non-negative multiplicative function such that, for some A and B, $\sum_{p\le y} f(p)\log p \le Ay \quad (y\ge 0)$, $\sum_p\sum_{\nu\ge 2}\frac{f(p^\nu)}{p^\nu}\log p^\nu \le B$. Then, for $x>1$, $\sum_{n\le x} f(n) \le (A+B+1)\frac{x}{\log x}\sum_{n\le x}\frac{f(n)}{n}$

See also Section 4.6, and for instance Theorem 4.22, of *Bordell:cite:`Bordelles12*. In particular, in case a further condition is assumed, we have Theorem 4.28 of *Bordell:cite:`Bordelles12* at our disposal.

Theorem (2012)

Let f be a non-negative multiplicative function such that, for every prime p and every non-negative power a the condition $f(p^{a+1})\ge f(p^a)$ holds, we have for $x\ge 1$ $\sum_{n\le x} f(n) \le x\prod_{p\le x}\left(1-\frac{1}{p}\right)\left(1+\sum_{a\ge 1}\frac{f(p^a)}{p^a}\right)$.

The next lemma is handy to remove coprimality conditions. It originates from [van Lint and

Richert, 1965].

Theorem (1965)

Let f be a non-negative multiplicative function and let d be a positive integer. We have for $x \geq 0$ $\sum_{n\leq x}\mu^2(n)f(n) \leq \prod_{p|d}(1+f(p)) \sum_{\substack{n\leq x,\\(n,d)=1}} \mu^2(n)f(n) \leq \sum_{n\leq xd}\mu^2(n)f(n)$.

Though it is somewhat difficult to get, this lemma has been further generalized in Lemma 4.1 of [Ramaré, 2012].

11.3 3. Estimates of some special functions

[Cohen and Dress, 1988] contains the following Theorem.

Theorem (1988)

Let $R(x) = \sum_{n\leq x}\mu^2(n) - 6x/\pi^2$. We have $|R(x+y)-R(x)| \leq 1.6749\sqrt{y} + 0.6212x/y$ and $|R(x+y) - R(x)| \leq 0.7343y/x^{1/3} + 1.4327x^{1/3}$ for $x, y \geq 1$.

See also [Costa Pereira, 1989]. [Moser and MacLeod, 1966] and [Cohen *et al.*, 2007] contains:

Theorem (2008)

We have $|\sum_{n\leq x}\mu^2(n) - 6x/\pi^2| \leq 0.02767\sqrt{x}$ for $x \geq 438653$. One can replace $(0.02767, 438653)$ by $(0.036438, 82005)$, by $(0.1333, 1664)$, by $(1/2, 8)$ or by $(1, 1)$.

This estimate is (slightly) improved in Corollary 3.4 of [Ramaré, 2019] in the next one.

Theorem (2019)

When $x > 1$, we have $\sum_{n\leq x}\mu^2(n) = \frac{6}{\pi^2}x + O^*(1.06\sqrt{x/\log x})$.

Lemma 3.4 of [Ramaré, 2016] gives us:

Theorem (2013)

We have $\frac{6}{\pi^2}\log x + 0.578 \leq \sum_{n\leq x}\mu^2(n)/n \leq \frac{6}{\pi^2}\log x + 1.166$ for $x \geq 1$

When $x \geq 1000$, one can also replace the couple $(0.578, 1.166)$ by $(1.040, 1.048)$.

In Theorem 3.2 and Corollary 3.3 of [Ramaré, 2019], we find the next result.

Theorem (2019)

When $x \geq 3475$, we have $\sum_{n \leq x} \frac{\mu^2(n)}{n} = \frac{6}{\pi^2}\Big(\log x + 2\sum_{p \geq 2} \frac{\log p}{p^2-1} + \gamma\Big) + O^*(0.073/\sqrt{x})$. And when $x \geq 1665$, the error term may be (asymptotically) improved to $O^*(0.35/\sqrt{x \log x})$.

See also Lemma 1 of [Schoenfeld, 1969] for an earlier version.

Lemma 2 of [Riesel and Vaughan, 1983] contains the next evaluation.

Theorem (1983)

For $y \geq 1$, we have $\sum_{q \leq y} \mu^2(q) \prod_{\substack{p|q \\ p \neq 2}} \frac{2}{p-2} = \frac{1}{4} \prod_{p>2} \frac{(p-1)^2}{p(p-2)} \Big((\log y)^2 - A_3 \log y - A_4 + \mathcal{O}^*(1088/y^{1/3})\Big)$ where $A_3 = 6.023476\cdots$ and $A_4 = 1.114073\cdots$.

The main result [Berkane *et al.*, 2012] reads as follows.

Theorem (2012)

We define $\Delta(x) = \sum_{n \leq x} \tau(n) - x(\log x + 2\gamma - 1)$. We have

- When $x \geq 1$, we have $|\Delta(x)| \leq 0.961\, x^{1/2}$.
- When $x \geq 1\,981$, we have $|\Delta(x)| \leq 0.482\, x^{1/2}$.
- When $x \geq 5\,560$, we have $|\Delta(x)| \leq 0.397\, x^{1/2}$.
- When $x \geq 5$, we have $|\Delta(x)| \leq 0.764\, x^{1/3} \log x$.

For evaluation of the average of the divisor function on integers belonging to a fixed residue class modulo 6, see Corollary to Proposition 3.2 of [Deshouillers and Dress, 1988].

For more complicated sums and when x is large with respect to k, one can use [Mardjanichvili, 1939].

Theorem (1939)

Let k and ℓ be two positive integers. We have for any real number $x \geq 1$ $\sum_{m \leq x} \tau_k^\ell(m) \leq x \frac{k^\ell}{(k!)^{\frac{k^\ell-1}{k-1}}} (\log x + k^\ell - 1)^{k^\ell - 1}$.

See [Deshouillers and Dress, 1988] for some upper bounds linked with τ_3.

[Bordellès, 2002] contains the following bounds, better than the above when x is small with respect to k.

Theorem (2002)

Let $k \geq 1$ be a positive integer.

- When $x \geq 1$ is a real number, we have $\sum_{m \leq x} \tau_k(m) \leq x(\log x + \gamma + (1/x))^{k-1}$.
- When $x \geq 6$ is a real number, we have $\sum_{m \leq x} \tau_k(m) \leq 2x(\log x)^{k-1}$.

In [Cully-Hugill and Trudgian, 2021], we find the next result.

Theorem (2021)

For $x \geq 2$, we have $\sum_{n \leq x} \tau_4(n) = C_1 x(\log x)^3 + C_2 x(\log x)^2 + C_3 x(\log x + C_4 x + \mathcal{O}^*(4.48\, x^{3/4} \log x)$ where the constants $C_1 = 1/6$, $C_2 = 2\gamma - 1/2$, and C_3 and C_4 are the expected ones and may be expressed in terms of the Stieltjes constants γ_i.

They deduce for instance from this that $\sum_{n \leq x} \tau_4(n) \leq (1/3)x(\log x)^3$ when $x \geq 193$.

In the same paper, these authors also establish the next estimate.

Theorem (2021)

For $x \geq 2$, we have $\sum_{n \leq x} \tau(n)^2 = D_1 x(\log x)^3 + D_2 x(\log x)^2 + D_3 x(\log x + D_4 x + \mathcal{O}^*(9.73\, 4.48\, x^{3/4} \log x + 0.73\sqrt{x})$ where the constants $D_1 = 1/\pi^2$, and D_2, D_3 and D_4 are the expected ones and may be expressed in terms of usual constants.

They deduce for instance from this that $\sum_{n \leq x} \tau(n)^2 \leq (1/4)x(\log x)^3$ when $x \geq 433$ and that $\sum_{n \leq x} \tau(n)^2 \leq x(\log x)^3$ when $x \geq 7$.

In [Lapkova, 2016], we find the next result.

Theorem (2015)

Let b and c be two integers such that $\delta = b^2 - c$ is non-zero, square-free and not congruent to 1 modulo 4. Assume further that the function $n^2 + 2bn + c$ is positive and non-decreasing when $n \geq 1$.Then, for $N \geq 1$, we have $\sum_{n \leq N} \tau(n^2 + 2bn + c) \leq C_1 N \log N + C_2 + C_3$ where the constants C_1, C_2 and C_3 are defined as follows. We first define $\xi = \sqrt{1 + 2|b| + |c|}$ and $\kappa = \frac{4}{\pi^2}\sqrt{4|\delta|}(\log(4|\delta|) + 0.648)$. Then $C_1 = \frac{12}{\pi^2}(\log \kappa + 1), C_2 = 2\left[\kappa + (\log \kappa + 1)\left(\frac{6}{\pi^2}\log \xi + 1.166\right)\right], C_3 = 2\kappa(\max(|b|, |c|^{1/2}) + 1)$.

See [Lapkova, 2017] for the number of divisors of a reducible quadratic polynomial.

Evaluations of Lemma 4.3 of [Cipu, 2015] are improved in Lemma 12 of [Trudgian, 2015]. Only upper bounds are given, but the proof given there gives the lower bounds as well. This gives the first two estimates, while the third one comes from Lemma 4.3 of [Cipu, 2015].

Theorem (2015)

Let $x \geq 1$ be a real number. We have

- $0.786x - 0.3761 - 8.14x^{2/3} \leq \sum_{n\leq x} 2^{\omega(n)} - \frac{6}{\pi^2} x\log x \leq 0.787x - 0.3762 + 8.14x^{2/3}$
- $1.3947\log x + 0.4106 - 3.253x^{-1/3} \leq \sum_{n\leq x} \frac{2^{\omega(n)}}{n} - \frac{3}{\pi^2}(\log x)^2 \leq 1.3948\log x + 0.4107 + 3.253x^{-1/3}$,
- $\sum_{n\leq x} \frac{2^{\omega(2n-1)}}{2n-1} \leq \frac{3}{2\pi^2}(\log x)^2 + 3.123\log x + 3.569 + \frac{0.525}{x}$.

We take the next lemma from [Treviño, 2015], Lemma 2.

Theorem (2015)

Let $x \geq 1$ be a real number. We have $\sum_{n\leq x} \phi(n)/n \leq \frac{6}{\pi^2}x + \log x + 1$.

Lemma 3 of the same paper is as follows.

Theorem (2015)

Let $x \geq 1$ be a real number. We have $\sum_{n\leq x} n\phi(n) \leq \frac{2}{\pi^2}x^3 + \frac{1}{2}x^2\log x + x^2$.

Several estimates are proved in [P. and Ramaré, 2017], [Ramaré and Viswanadham, 2021] and in [Ramaré, 2019].

For instance Theorem 3.1 in the latter paper contains the following.

Theorem (2018)

Let $x \geq 1$ be a real number. We have $\sum_{n\leq x} \mu^2(n)/\phi(n) = \log x + c_0 + O^*(2.44/\sqrt{x})$ where $c_0 = \gamma + \sum_{p\geq 2} \frac{\log p}{p(p-1)}$. When $x > 1$, this O^* can be replaced by $O^*(11/\sqrt{x}\log x)$.

The function $\sum_{n\leq x} \mu^2(n)/\phi(n)$ has been the subject of several estimates, see for instance Lemma 7 of [Montgomery and Vaughan, 1973], Lemma 3.4-3.5 of [Ramaré, 1995], the earlier paper [Ward, 1927] and Lemma 4.5 of [Büthe, 2014] where the error term $O^*(58/\sqrt{x})$ is achieved. The constant c_0 is evaluated precisely in (2.11) of [Rosser and Schoenfeld, 1962].

11.4 4. Euler products

[Rosser and Schoenfeld, 1962] contains estimates regarding $\prod_p(1 - 1/p)$ and its inverse. In particular we find the next results therein.

Theorem (1962)

- When $x > 1$, we have $1 - \frac{1}{\log^2 x} \le e^{\gamma}(\log x)\prod_{p\le x}\left(1-\frac{1}{p}\right) \le 1 + \frac{1}{2\log^2 x}$.
- When $x > 1$, we have $1 - \frac{1}{2\log^2 x} \le e^{-\gamma}\prod_{p\le x}\left(1-\frac{1}{p}\right)^{-1}/\log x \le 1 + \frac{1}{\log^2 x}$.

Several other estimates are proven. In [Dusart, 2018], it is proved that

Theorem (2016)

- When $x > 2\,278\,382$, we have $1 - \frac{1}{5\log^3 x} \le e^{\gamma}(\log x)\prod_{p\le x}\left(1-\frac{1}{p}\right) \le 1 + \frac{1}{5\log^3 x}$.
- When $x > 2\,278\,382$, we have $1 - \frac{1}{5\log^3 x} \le e^{-\gamma}\prod_{p\le x}\left(1-\frac{1}{p}\right)^{-1}/\log x \le 1 + \frac{1}{5\log^3 x}$.

In [Bordellés, 2005], the reader will find explicit upper bounds for $\prod_{\substack{p\le x,\\ p\equiv a[q]}}\left(1-\frac{1}{p}\right)^{-1}$.

Theorem 5 of [Mawia, 2017] contains the next result.

Theorem (2017)

Let ϵ be a complex number such that $|\epsilon| \le 2$. When $x \ge \exp(22)$, we have $\prod_{p\le x}\left(1+\frac{\epsilon}{p}\right) = e^{\gamma(\epsilon)+\epsilon B}(\log x)^{\epsilon}\left\{1+O^*\left(\frac{0.841}{\log^3 x}\right)\right\}$ where $\gamma(\epsilon) = \sum_{p\ge 2}\sum_{n\ge 2}(-1)^{n+1}\frac{\epsilon^n}{np^n}$ and $B = \gamma + \sum_{p\ge 2}\big(\log(1-1/p) + (1/p)\big)$.

Equation (2.2) of [Rosser and Schoenfeld, 1962] gives an approximate value for B.

12

Integer Points near Smooth Plane Curves

In what follows, $N \geqslant 1$ is an arbitrary large integer, $\delta \in \left(0, \frac{1}{2}\right)$ and if $f : [N, 2N] \longrightarrow \mathbb{R}$ is any positive function, then let $\mathcal{R}(f, N, \delta)$ be the number of integers $n \in [N, 2N]$ such that $\|f(n)\| < \delta$, where as usual $\|x\|$ denotes the distance from $x \in \mathbb{R}$ to its nearest integer. Note that, since δ is very small,:math:*mathcal {R}(f,N,delta)`roughly counts the number of integer points very close to the arc :math:`y = f(x)* with $N \leq x \leq 2N$. Hence the trivial estimate is given by $\mathcal{R}(f, N, \delta) \leq N + 1$.

The number $\mathcal{R}(f, N, \delta)$ arises fairly naturally in a large collection of problems in number theory, e.g. [Filaseta, 1990], [Filaseta and Trifonov, 1996], [Huxley, 1996], [Huxley and Sargos, 1995], [Huxley and Sargos, 2006], [Huxley and Trifonov, 1996] and [Huxley, 2007]. We deal with either getting an asymptotic formula of the shape

$\mathcal{R}(f, N, \delta) = N\delta +$ Error terms

where the remainder terms depend on the derivatives of f but not on δ, or finding an upper bound for $\mathcal{R}(f, N, \delta)$ as accurate as possible.

12.1 1. Bounds using elementary methods

The basic result of the theory is well-known and may be found in [Vinogradov, 2004]. The proof follows from a clever use of the mean-value theorem (see Theorem~5.6 of *:cite:Bordelles12* for instance).

Theorem (First derivative test)

Let $f \in C^1[N, 2N]$ such that there exist $\lambda_1 > 0$ and $c_1 \geq 1$ such that, for all $x \in [N, 2N]$, we have $\lambda_1 \leq |f'(x)| \leq c_1\lambda_1$. Then $\mathcal{R}(f, N, \delta) \leq 2c_1N\lambda_1 + 4c_1N\delta + \dfrac{2\delta}{\lambda_1} + 1$.

This result is useful when λ_1 is very small, so that the condition is in general too restrictive in the applications. Using a rather neat combinatorial trick, [Huxley, 1996] succeeded in passing from the first derivative to the second derivative. This reduction step enables him to apply this theorem to a function being approximatively of the same order of magnitude as f'. This provides the following useful result.

Theorem (Second derivative test)

Let $f \in C^2[N,2N]$ such that there exist $\lambda_2 > 0$ and $c_2 \geq 1$: *math* such that, for all $x \in [N,2N]$, we have $\lambda_2 \leq |f''(x)| \leq c_2\lambda_2$ and $N\lambda_2 \geq c_2^{-1}$. Then $\mathcal{R}(f,N,\delta) \leq 6\left\{(3c_2)^{1/3} N\lambda_2^{1/3} + (12c_2)^{1/2} N\delta^{1/2} + 1\right\}$.

Both hypotheses above are often satisfied in practice, so that this result may be considered as the first useful tool of the theory. A proof of this Theorem may be found in Theorem 5.8 of [Bordellès, 2012].

Using a k-th version of Huxley's reduction principle may allow us to generalize the above results. A better way is to split the integer points into two classes, namely the major arcs in which the points belong to a same algebraic curve of degree $\leq k-1$, and the minor arcs. The points coming from the minor arcs are treated by divided differences techniques, generalizing the proof of both theorems above and, by a careful analysis of the points belonging to major arcs, [Huxley and Sargos, 1995] and [Huxley and Sargos, 2006] succeeded in proving the following fundamental result. A proof of an explicit version may be found in Theorem 5.12 of [Bordellès, 2012].

Theorem (k th derivative test)

Let $k \geq 3$ be an integer and $f \in C^k[N,2N]$ such that there exist $\lambda_k > 0$ and $c_k \geq 1$ such that, for all $x \in [N,2N]$, we have $\lambda_k \leq \left|f^{(k)}(x)\right| \leq c_k\lambda_k$. Let $\delta \in \left(0,\frac{1}{4}\right)$. Then $\mathcal{R}(f,N,\delta) \leq \alpha_k N\lambda_k^{\frac{2}{k(k+1)}} + \beta_k N\delta^{\frac{2}{k(k-1)}} + 8k^3\left(\frac{\delta}{\lambda_k}\right)^{1/k} + 2k^2\left(5e^3+1\right)$ where $\alpha_k = 2k^2 c_k^{\frac{2}{k(k+1)}}$ and $\beta_k = 4k^2\left(5e^3 c_k^{\frac{2}{k(k-1)}} + 1\right)$.

12.2 2. Bounds using exponential sums techniques

The next result leads us to estimate $\mathcal{R}(f,N,\delta)$ with the help of exponential sums (see [Graham and Kolesnik, 1991] for instance), which have been extensively studied in the 20th century by many specialists, such as van der Corput, Weyl or Vinogradov. Nevertheless, even using the finest exponent pairs to date, the result generally does not significantly improve on the previous estimates seen above. A simple proof of the following inequality may be found in [Filaseta, 1990].

Theorem (kth derivative test)

Let $f : [N,2N] \longrightarrow \mathbb{R}$ be any function and $\delta \in \left(0,\frac{1}{4}\right)$. Set $K = \lfloor (8\delta)^{-1} \rfloor + 1$. Then, for any positive integer $H \leq K$, we have $\mathcal{R}(f,N,\delta) \leq \frac{4N}{H} + \frac{4}{H}\sum_{h=1}^{H}\left|\sum_{N\leq n\leq 2N} e(hf(n))\right|$.

12.3 3. Integer points on curves

This last part is somewhat out of the scope of the TME-EMT project, but may help the reader in orienting him/herself in the litterature.

When $\delta \longrightarrow 0$, we are led to counting the number of integer points lying on curves, and we denote this number by $\mathcal{R}(f,N,0)$. This problem goes back to Jarnik [Jarník, 1925] who proved that a strictly convex arc $y=f(x)$ with length L has at most

$\leq \frac{3}{(2\pi)^{1/3}} L^{2/3} + O\left(L^{1/3}\right)$ integer points and this is a nearly best possible result under the sole hypothesis of convexity. However, [Swinnerton-Dyer, 1974] proved that if $f \in C^3[0,N]$ is such that $|f(x)| \leq N$ and $f'''(x) \neq 0$ for all $x \in [0,N]$, then the number of integer points on the arc $y=f(x)$ with $0 \leq x \leq N$ is $\ll N^{3/5+\varepsilon}$. This result was later generalized by [Bombieri and Pila, 1989] who showed among other things the following estimate.

Theorem (1989)

Let $N \geq 1$, $k \geq 4$ be integers and define $K = \binom{k+2}{2}$. Let $\mathcal{I}$ be an interval with length N and $f \in C^K(\mathcal{I})$ satisfying $|f'(x)| \leq 1$, $f''(x) > 0$ and such that the number of solutions of the equation $f^{(K)}(x) = 0$ is $\leq m$. Then there exists a constant $c_0 = c_0(k) > 0$ such that $\mathcal{R}(f,N,0) \leq c_0(m+1)N^{1/2+3/(k+3)}$.

13 Explicit pointwise upper bounds for some arithmetic functions

The following bounds may be useful is applications.

From [Robin, 1983]:

Theorem (1983)

For any integer $n \geq 3$, the number of prime divisors $\omega(n)$ of n satisfies: $\omega(n) \leq 1.3841 \dfrac{\log n}{\log \log n}$.

From [Nicolas and Robin, 1983]:

Theorem (1983)

For any integer $n \geq 3$, the number $\tau(n)$ of divisors of n satisfies: $\tau(n) \leq n^{1.538 \log 2 / \log \log n}$.

From page 51 of [Robin, 1983]:

Theorem (1983)

For any integer $n \geq 3$, we have $\tau_3(n) \leq n^{1.59141 \log 3 / \log \log n}$ where $\tau_3(n)$ is the number of triples (d_1, d_2, d_3) such that $d_1 d_2 d_3 = n$.

The PhD memoir [Duras, 1993] contains result concerning the maximum of $\tau_k(n)$, i.e. the number of k-tuples $(d_1, d_2, \ldots, d_k)$ such that $d_1 d_2 \cdots d_k = n$, when $3 \leq k \leq 25$.

From [Duras *et al.*, 1999]:

Theorem (1999)

For any integer $n \geq 1$, any real number $s > 1$ and any integer $k \geq 1$, we have $\tau_k(n) \leq n^s \zeta(s)^{k-1}$ where $\tau_k(n)$ is the number of k-tuples $(d_1, d_2, \cdots, d_k)$ such that $d_1 d_2 \cdots d_k = n$.

The same paper also announces the bound for $n \geq 3$ and $k \geq 2$

$\tau_k(n) \leq n^{a_k \log k/\log\log k}$ where $a_k = 1.53797 \log k/\log 2$ but the proof never appeared.

From [Nicolas, 2008]:

Theorem (2008)

For any integer $n \geq 3$, we have $\sigma(n) \leq 2.59791\, n \log\log(3\tau(n))$, $\sigma(n) \leq n\{e^\gamma \log\log(e\tau(n)) + \log\log\log(e^e\tau(n)) + 0.9415\}$.

The first estimate above is a slight improvement of the bound

$\sigma(n) \leq 2.59n \log\log n \quad (n \geq 7)$ obtained in [Ivić, 1977]. In this same paper, the author proves that

$\sigma^*(n) \leq \frac{28}{15} n \log\log n \quad (n \geq 31)$ where $\sigma^*(n)$ is the sum of the unitary divisors of n, i.e. divisors d of n that are such that d and n/d are coprime.

In [Eum and Koo, 2015] we find the next estimate

Theorem (2015)

For any integer $n \geq 21$, we have $\sigma(n) \leq \frac{3}{4} e^\gamma n \log\log n$.

Further estimates restricted to some sets of integers may be found in this paper as well as in [Washington and Yang, 2021].

On this subject, the readers may consult the web site

The sum of divisors function and the Riemann hypothesis. .

14

Explicit bounds for class numbers

Let K be a number field of degree $n \geq 2$, signature (r_1, r_2), absolute value of discriminant d_K, class number h_K, regulator $\mathcal{R}_K$ and w_K the number of roots of unity in K. We further denote by κ_K the residue at $s = 1$ of the Dedekind zeta-function $\zeta_K(s)$ attached to K.

Estimating h_K is a long-standing problem in algebraic number theory.

14.1 1. Majorising $h_K \mathcal{R}_K$

One of the classic way is the use of the so-called analytic class number formula stating that

$h_K \mathcal{R}_K = \frac{w_K \sqrt{d_K}}{2^{r_1}(2\pi)^{r_2}} \kappa_K$ and to use Hecke's integral representation of the Dedekind zeta function to bound κ_K. This is done in [Louboutin, 2000] and in [Louboutin, 2001] with additional properties of log-convexity of some functions related to ζ_K and enabled Louboutin to reach the following bound:

$h_K \mathcal{R}_K \leq \frac{w_K}{2} \left(\frac{2}{\pi}\right)^{r_2} \left(\frac{e \log d_K}{4n-4}\right)^{n-1} \sqrt{d_K}$. Furthermore, if $\zeta_K(\beta) = 0$ for some $\frac{1}{2} \leq \beta < 1$, then we have

$h_K \mathcal{R}_K \leq (1-\beta) w_K \left(\frac{2}{\pi}\right)^{r_2} \left(\frac{e \log d_K}{4n}\right)^{n} \sqrt{d_K}$. When K is abelian, then the residue κ_K may be expressed as a product of values at $s = 1$ of L-functions associated to primitive Dirichlet characters attached to K. On using estimates for such L-functions from [Ramaré, 2001], we get for instance

$h_K \mathcal{R}_K \leq \frac{w_K}{2} \left(\frac{2}{\pi}\right)^{r_2} \left(\frac{\log d_K}{4n-4} + \frac{5 - \log 36}{4}\right)^{n-1} \sqrt{d_K}$. Note that the constant $\frac{1}{4}(5 - \log 36) = 0.354\cdots$ can be improved upon in many cases. For instance, when K is abelian and totally real (i.e. $r_2 = 0$), a result fromd [Ramaré, 2001] implies that the constant may be replaced by 0, so that

$h_K \mathcal{R}_K \leq \left(\frac{\log d_K}{4n-4}\right)^{n-1} \sqrt{d_K}$.

14.2 2. Majorising h_K

One may also estimate h_K alone, without any contamination by the regulator since this contamination is often difficult to control, see [Pohst and Zassenhaus, 1989].

In this case, one rather uses explicit bounds for the Piltz-Dirichlet divisor functions τ_n (see [Bordellès, 2002] and [Bordellès, 2006]) and get

$h_K \leq \frac{M_K}{(n-1)!}\left(\frac{\log(M_K^2 d_K)}{2}+n-2\right)^{n-1}\sqrt{d_K}$ as soon as

$n \geq 3, \quad d_K \geq 139 M_K^{-2} \quad \text{where} \quad M_K = (4/\pi)^{r_2} n!/n^n$. The constant M_K is known as the Minkowski constant of K.

In [Cully-Hugill and Trudgian, 2021] we find the following.

Theorem (2021)

Ler K be a quartic number field with class number h_K and Minkowski bound b. Then if $b \geq 193$, we have $h_K \leq (1/3)x(\log x)^3$.

14.3 3. Using the influence of the small primes

It is explained in [Louboutin, 2005] how the behavior of certain small primes may subtantially improve on the previous bounds. To make things more significant, define, for a rational prime p,

$\Pi_K(p) = \prod_{\mathfrak{p}|p}\left(1-\frac{1}{\mathcal{N}_K(\mathfrak{p})}\right)^{-1}$. From [Louboutin, 2005], we have among other things

$h_K\mathcal{R}_K \ \leq \ \frac{w_K}{2}\left(\frac{2}{\pi}\right)^{r_2}\frac{\Pi_K(2)}{\Pi_{\mathbb{Q}}(2)^n}\left(\frac{e\log d_K}{4n-4}\times e^{n\log 4/\log d_K}\right)^{n-1}\sqrt{d_K}$ where K is any number field of degree $n \geq 3$. In particular, when 2 is inert in K, then

$h_K\mathcal{R}_K \leq \frac{w_K}{2(2^n-1)}\left(\frac{2}{\pi}\right)^{r_2}\left(\frac{e\log d_K}{4n-4}\times e^{n\log 4/\log d_K}\right)^{n-1}\sqrt{d_K}.$

14.4 4. The h_K^- of CM-fields

Let K be here a CM-field of degree $2n > 2$, i.e. a totally complex quadratic extension K of its maximal totally real subfield K^+. it is well known that h_{K^+} divides h_K. The quotient is denoted by h_K^- and is called the relative class number of K. The analytic class number formula yields

$h_K^- \ = \ \frac{Q_K w_K}{(2\pi)^n}\left(\frac{d_K}{d_{K^+}}\right)^{1/2}\frac{\kappa_K}{\kappa_{K^+}} \ = \ \frac{Q_K w_K}{(2\pi)^n}\left(\frac{d_K}{d_{K^+}}\right)^{1/2} L(1,\chi)$ where χ is the quadratic character of degree 1 attached to the extension K/K^+ and $Q_K \in \{1,2\}$ is the Hasse unit index of K. Here are three results originating in this formula.

From [Louboutin, 2000]:

Theorem (2000)

We have $h_K^- \le 2Q_K w_K \left(\frac{d_K}{d_{K^+}}\right)^{1/2} \left(\frac{e\log(d_K/d_{K^+})}{4\pi n}\right)^n$.

From [Louboutin, 2003]:

Theorem (2003)

Assume that $(\zeta_K/\zeta_{K^+})(\sigma) \ge 0$ whenever $0 < \sigma < 1$. Then we have $h_K^- \ge \frac{Q_K w_K}{\pi e \log d_K} \left(\frac{d_K}{d_{K^+}}\right)^{1/2} \left(\frac{n-1}{\pi e \log d_K}\right)^{n-1}$.

Again from [Louboutin, 2003]:

Theorem (2003)

Let $c = 6 - 4\sqrt{2} = 0.3431\cdots$. Assume that $d_K \ge 2800^n$ and that either K does not contain any imaginary quadratic subfield, or that the real zeros in the range $1 - \frac{c}{\log d_N} \le \sigma < 1$ of the Dedekind zeta-functions of the imaginary quadratic subfields of K are nor zeros of $\zeta_K(s)$, where N is the normal closure of K. Then we have $h_K^- \ge \frac{cQ_K w_K}{4ne^{c/2}[N:\mathbb{Q}]} \left(\frac{d_K}{d_{K^+}}\right)^{1/2} \left(\frac{n}{\pi e \log d_K}\right)^n$.

And a third result from [Louboutin, 2003]:

Theorem (2003)

Assume $n > 2$, $d_K > 2800^n$ and that K contains an imaginary quadratic subfield F such that $\zeta_F(\beta) = \zeta_K(\beta) = 0$ for some β satisfying $1 - \frac{2}{\log d_K} \le \beta < 1$. Then we have $h_K^- \ge \frac{6}{(\pi e)^2} \left(\frac{d_K}{d_{K^+}}\right)^{1/2-1/n} \left(\frac{n}{\pi e \log d_K}\right)^{n-1}$.

15

Sieve bounds

15.1 1. Some upper bounds

Theorem 2 of [Montgomery and Vaughan, 1973] contains the following explicit version of the Brun-Tichmarsh Theorem.

Theorem (1973)

Let x and y be positive real numbers, and let k and ℓ be relatively prime positive integers. Then $\pi(x+y;k,\ell)-\pi(x;k,\ell)\le \frac{2y}{\phi(k)\log(y/k)}$ provided only that $y>k$.

Here as usual, we have used the notation

$\pi(z;k,\ell)=\sum_{\substack{p\le z,\\ p\equiv \ell[k]}}1$, i.e. the number of primes up to z that are coprime to ℓ modulo k. See [Büthe, 2014] for a generic weighted version of this inequality.

Lemma 14 of [Ramaré and Viswanadham, 2021], the following extension of the above is proved.

Theorem (2021)

Let $x\ge y>k\ge 1$ be positive real numbers, k being an integer. Then $\sum_{\substack{x<m\le x+y\\ m\equiv a[k]}}\frac{\Lambda(m)}{\log m}<\frac{2y}{\phi(k)\log(y/k)}$.

And in Lemma 15 of the same paper, we find the next estimate.

Theorem (2021)

Let $x \geq \max(121, k^3)$. Then $\sum_{\substack{x<m\leq 2x\\ m\equiv a[k]}} \Lambda(m) < \frac{9}{2}\frac{x}{\phi(k)}$.

Here is a bound concerning a sieve of dimension 2 proved by [Siebert, 1976].

Theorem (1976)

Let a and b be coprime integers with $2|ab$. Then we have, for $x > 1$, $\sum_{\substack{p\leq x,\\ ap+b \text{ prime}}} 1 \leq 16\omega\frac{x}{(\log x)^2}\prod_{\substack{p|ab,\\ p>2}}\frac{p-1}{p-2} \qquad \omega = \prod_{p>2}(1-(p-1)^{-2})$.

This is improved for large values in Lemma 4 of [Riesel and Vaughan, 1983].

Theorem (1983)

Let a and b be coprime integers with $2|ab$. Then we have, for $x \geq e^L$, $\sum_{\substack{p\leq x,\\ ap+b \text{ prime}}} 1 \leq \left(\frac{16\omega\, x}{(\log x)(A+\log x)} - 100\sqrt{x}\right)\prod_{\substack{p|ab,\\ p>2}}\frac{p-1}{p-2} \qquad \omega = \prod_{p>2}(1-(p-1)^{-2})$ and where

L: 24 25 26 27 28 29 31 34 42 60 690

A: 0 1 2 3 4 5 6 7 8 8.3 8.45

15.2 2. Density estimates

In Theorem 1, page 52 of [Greaves, 2001], we find the next widely applicable estimate.

Theorem (2022)

Let κ be a non-negative function on the primes such that $\kappa(p) < p$. Assume there is a constant B such that $\sum_{p<z} \frac{\kappa(p)\log p}{p} \leq B \log z$ for some $z \geq 2$. Then, when $s \geq 2B$, we have

$$\sum_{d \leq z^{s/2}} \mu^2(d) \prod_{p|d} \frac{\kappa(p)}{p-\kappa(p)} \geq \Big(1 - \exp -\big(\frac{s}{2}\log\frac{s}{2B} - \frac{s}{2} + B\big)\Big) \prod_{p<z}\Big(1 - \frac{\kappa(p)}{p}\Big)^{-1}.$$

See also herefootnote{url{Articles/Art10.html#asy}}.

15.3 3. Combinatorial sieve estimates

The combinatorial sieve is known to be difficult from an explicit viewpoint. For the linear sieve, the reader may look at Chapter 9, Theorem 9.7 and 9.8 from [Nathanson, 1996].

15.4 4. Integers free of small prime factors

In [Fan, 2022], the following neat estimate is proved.

Theorem (2022)

Let $\Phi(x, z)$ be the number of integers $\leq x$ that do not have any prime factors $\leq z$. We have $\Phi(x, z) \leq \frac{x}{\log z}, \quad (1 < z \leq x)$.

16 Character sums

16.1 1. Explicit Polya-Vinogradov inequalities

The main Theorem of [Qiu, 1991] implies the following result.

Theorem (1991)

For χ a primitive character to the modulus $q > 1$, we have $\left|\sum_{a=M+1}^{M+N} \chi(a)\right| \leq \frac{4}{\pi^2}\sqrt{q}\log q + 0.38\sqrt{q} + \frac{0.637}{\sqrt{q}}$.

When χ is not especially primitive, but is still non-principal, we have $\left|\sum_{a=M+1}^{M+N} \chi(a)\right| \leq \frac{8\sqrt{6}}{3\pi^2}\sqrt{q}\log q + 0.63\sqrt{q} + \frac{1.05}{\sqrt{q}}$.

This was improved later by [Bachman and Rachakonda, 2001] into the following.

Theorem (2001)

For χ a non-principal character to the modulus $q > 1$, we have $\left|\sum_{a=M+1}^{M+N} \chi(a)\right| \leq \frac{1}{3\log 3}\sqrt{q}\log q + 6.5\sqrt{q}$.

These results are superseded by [Frolenkov, 2011] and more recently by [Frolenkov and Soundararajan, 2013] into the following.

Theorem (2013)

For χ a non-principal character to the modulus $q \geq 1000$, we have $\left|\sum_{a=M+1}^{M+N} \chi(a)\right| \leq \frac{1}{\pi\sqrt{2}}\sqrt{q}(\log q + 6) + \sqrt{q}$.

In the same paper they improve upon estimates of [Pomerance, 2011] and get the following.

Theorem (2013)

For χ a primitive character to the modulus $q \geq 1200$, we have $\max_{M,N}\left|\sum_{a=M+1}^{M+N}\chi(a)\right| \leq \begin{cases} \frac{2}{\pi^2}\sqrt{q}\log q+\sqrt{q}, & \chi \text{ even}, \\ \frac{1}{2\pi}\sqrt{q}\log q+\sqrt{q}, & \chi \text{ odd}. \end{cases}$ This latter estimates holds as soon as $q \geq 40$.

In case χ odd, the constant $1/(2\pi)$ has already been asymptotically obtained in [Landau, 1918]. When χ is odd and $M=1$, the best asymptotical constant before 2020 was $1/(3\pi)$ from Theorem 7 of [Granville and Soundararajan, 2007], In case χ even, we have

$\max_{M,N}\left|\sum_{a=M}^{N}\chi(a)\right| = 2\max_{N}\left|\sum_{a=1}^{N}\chi(a)\right|$. (The LHS is always less than the RHS. Equality is then easily proved). The asymptotical best constant in 2007 was $23/(35\pi\sqrt{3})$ from Theorem 7 of [Granville and Soundararajan, 2007].

These results are improved upon for large values squarefree values of q in [Bordignon and Kerr, 2020] by a different method into the following.

Theorem (2020)

For χ a primitive character to the squarefree modulus $q \geq \exp(1088\ell^2)$, we have $\max_{N}\left|\sum_{a=1}^{N}\chi(a)\right| \leq \begin{cases} \frac{2}{\pi^2}\sqrt{q}\left(\frac{1}{4}+\frac{1}{4\ell}\right)\log q+\left(49+\frac{1}{1088\ell}\right)\sqrt{q}, & \chi \text{ even}, \\ \frac{1}{2\pi}\left(\frac{1}{2}+\frac{1}{2\ell}\right)\sqrt{q}\log q+\left(49+\frac{1}{1088\ell}\right)\sqrt{q}, & \chi \text{ odd}. \end{cases}$ This latter estimates holds as soon as $q \geq 40$.

Corresponding estimates when q is not squarefree are proved in [Bordignon, 2021], the saving $1/4$ being slightly degraded to $3/8$.

16.2 2. Burgess type estimates

The following from [Treviño, 2015] is an explicit version of Burgess with the only restriction being $p \geq 10^7$.

Theorem (2015)

Let p be a prime such that $p \geq 10^7$. Let χ be a non-principal character mod p. Let r be a positive integer, and let M and N be non-negative integers with $N \geq 1$. Then $\left|\sum_{a=M+1}^{M+N}\chi(a)\right| \leq 2.74N^{1-\frac{1}{r}}p^{\frac{r+1}{4r^2}}(\log p)^{\frac{1}{r}}$.

From the same paper, we get the following more specific result.

Theorem (2015)

Let p be a prime. Let χ be a non-principal character mod p. Let M and N be non-negative integers with $N \geq 1$, let $2 \leq r \leq 10$ be a positive integer, and let p_0 be a positive real number. Then for $p \geq p_0$, there exists $c_1(r)$, a constant depending on r and p_0 such that $\left|\sum_{a=M+1}^{M+N} \chi(a)\right| \leq c_1(r) N^{1-\frac{1}{r}} p^{\frac{r+1}{4r^2}} (\log p)^{\frac{1}{r}}$ where $c_1(r)$ is given by

r	$p_0 = 10^7$	$p_0 = 10^{10}$	$p_0 = 10^{20}$
2	2.7381	2.5173	2.3549
3	2.0197	1.7385	1.3695
4	1.7308	1.5151	1.3104
5	1.6107	1.4572	1.2987
6	1.5482	1.4274	1.2901
7	1.5052	1.4042	1.2813
8	1.4703	1.3846	1.2729
9	1.4411	1.3662	1.2641
10	1.4160	1.3495	1.2562

We can get a smaller exponent on log if we restrict the range of N or if we have $r \geq 3$.

Theorem (2015)

Let p be a prime. Let χ be a non-principal character mod p. Let M and N be non-negative integers with $1 \leq N \leq 2p^{\frac{1}{2}+\frac{1}{4r}}$ or $r \geq 3$. Let $r \leq 10$ be a positive integer, and let p_0 be a positive real number. Then for $p \geq p_0$, there exists $c_2(r)$, a constant depending on r and p_0 such that $\left|\sum_{a=M+1}^{M+N} \chi(a)\right| \leq c_2(r) N^{1-\frac{1}{r}} p^{\frac{r+1}{4r^2}} (\log p)^{\frac{1}{2r}}$, where $c_2(r)$ is given by

r	$p_0 = 10^7$	$p_0 = 10^{10}$	$p_0 = 10^{20}$
2	3.7451	3.5700	3.5341
3	2.7436	2.5814	2.4936
4	2.3200	2.1901	2.1071
5	2.0881	1.9831	1.9037
6	1.9373	1.8504	1.7748
7	1.8293	1.7559	1.6843
8	1.7461	1.6836	1.6167
9	1.6802	1.6262	1.5638
10	1.6260	1.5786	1.5210

Kevin McGown in [McGown, 2012] has slightly worse constants in a slightly larger range of N for smaller values of p.

Theorem (2012)

Let $p \geq 2{\cdot}10^4$ be a prime number. Let M and N be non-negative integers with $1 \leq N \leq 4p^{\frac{1}{2}+\frac{1}{4r}}$. Suppose χ is a non-principal character mod p. Then there exists a computable constant $C(r)$ such that $\left|\sum_{a=M+1}^{M+N}\chi(a)\right| \leq C(r)N^{1-\frac{1}{r}}p^{\frac{r+1}{4r^2}}(\log p)^{\frac{1}{2r}}$, where $C(r)$ is given by

r	$C(r)$	r	$C(r)$
2	10.0366	9	2.1467
3	4.9539	10	2.0492
4	3.6493	11	1.9712
5	3.0356	12	1.9073
6	2.6765	13	1.8540
7	2.4400	14	1.8088
8	2.2721	15	1.7700

If the character is quadratic (and with a more restrictive range), we have slightly stronger results due to Booker in [Booker, 2006].

Theorem (2006)

Let $p > 10^{20}$ be a prime number with $p \equiv 1 \pmod 4$. Let $r \in \{2, 3, 4, \ldots, 15\}$. Let M and N be real numbers such that $0 < M, N \leq 2\sqrt{p}$. Let χ be a non-principal quadratic character mod p. Then $\left|\sum_{a=M+1}^{M+N}\chi(a)\right| \leq \alpha(r)N^{1-\frac{1}{r}}p^{\frac{r+1}{4r^2}}(\log p + \beta(r))^{\frac{1}{2r}}$, where $\alpha(r)$ and $\beta(r)$ are given by

r	$\alpha(r)$	$\beta(r)$	r	$\alpha(r)$	$\beta(r)$
2	1.8221	8.9077	9	1.4548	0.0085
3	1.8000	5.3948	10	1.4231	-0.4106
4	1.7263	3.6658	11	1.3958	-0.7848
5	1.6526	2.5405	12	1.3721	-1.1232
6	1.5892	1.7059	13	1.3512	-1.4323
7	1.5363	1.0405	14	1.3328	-1.7169
8	1.4921	0.4856	15	1.3164	-1.9808

Concerning composite moduli, we have the next result in [Jain-Sharma *et al.*, 2021] .

Theorem (2021)

Let χ be a primitive character with modulus $q \geq e^{e^{9.594}}$. Then for $N \leq q^{5/8}$, we have $\left|\sum_{a=M+1}^{M+N}\chi(a)\right| \leq 9.07\sqrt{N}q^{3/16}(\log q)^{1/4}\left(2^{\omega(q)}d(q)\right)^{3/4}\sqrt{\frac{q}{\varphi(q)}}$.

17

Tools on Fourier transforms

17.1 1. The large sieve inequality

The best version of the large sieve inequality from [Montgomery and Vaughan, 1974] and [Montgomery and Vaughan, 1973] (obtained at the same time by A. Selberg) is as follows.

Theorem (1974)

Let M and $N \geq 1$ be two real numbers. Let X be a set of points of $[0,1)$ such that $\min_{\substack{x,y\in X\\ x\neq y}} \min_{k\in\mathbb{Z}} |x-y+k| \geq \delta > 0$. Then, for any sequence of complex numbers $(a_n)_{M<n\leq M+N}$, we have $\sum_{x\in X}\left|\sum_{M<n\leq M+N} a_n \exp(2i\pi nx)\right|^2 \leq \sum_{M<n\leq M+N} |a_n|^2(N-1+\delta^{-1})$.

It is very often used with part of the Farey dissection.

Theorem (1974)

Let M and $N \geq 1$ be two real numbers. Let $Q \geq 1$ be a real parameter. For any sequence of complex numbers $(a_n)_{M<n\leq M+N}$, we have $\sum_{q\in Q}\sum_{\substack{a \mod q,\\ (a,q)=1}}\left|\sum_{M<n\leq M+N} a_n \exp(2i\pi na/q)\right|^2 \leq \sum_{M<n\leq M+N} |a_n|^2(N-1+Q^2)$.

The summation over a runs over all invertible (also called reduced) classes a modulo q.

The weighted large sieve inequality from Theorem 1 in [Montgomery and Vaughan, 1974] reads as follows.

Theorem (1974)

Let M and $N \geq 1$ be two real numbers. Let X be a set of points of $[0,1)$. Define $\delta(x) = \min_{\substack{y\in X\\ y\neq x}} \min_{k\in\mathbb{Z}} |x-y+k|$. Then, for any sequence of complex numbers $(a_n)_{M<n\leq M+N}$, we have

$$\sum_{x\in X}\left(N+c\delta(x)^{-1}\right)^{-1}\left|\sum_{M<n\leq M+N} a_n \exp(2i\pi nx)\right|^2 \leq \sum_{M<n\leq M+N} |a_n|^2 \text{ for } c=3/2.$$

It is expected that one can reduce the constant $c = 3/2$ to 1. In this direction, we find in [Preissmann, 1984] the next result.

Theorem (1984)

We may take $c = \sqrt{1+\frac{2}{3}\sqrt{\frac{6}{5}}} = 1.3154\cdots < 4/3$ in the previous theorem.

See [Yangjit, 2023] for a discussion on this topic.

18

Tools on Mellin transforms

18.1 1. Explicit truncated Perron formula

Here is Theorem 7.1 of [Ramaré, 2007].

Theorem (2007)

Let $F(z) = \sum_n a_n/n^z$ be a Dirichlet series that converges absolutely for $\Re z > \kappa_a$, and let $\kappa > 0$ be strictly larger than κ_a. For $x \geq 1$ and $T \geq 1$, we have $\sum_{n \leq x} a_n = \frac{1}{2i\pi}\int_{\kappa-iT}^{\kappa+iT} F(z)\frac{x^z dz}{z} + \mathcal{O}^*\left(\int_{1/T}^{\infty} \sum_{|\log(x/n)| \leq u} \frac{|a_n|}{n^\kappa}\frac{2x^\kappa du}{Tu^2}\right)$.

See [Ramaré, 2016] for different versions.

18.2 2. L^2-means

We start with a majorant principle taken for instance from [Montgomery, 1994], chapter 7, Theorem 3.

Theorem

Let $\lambda_1, \cdots, \lambda_N$ be N real numbers, and suppose that $|a_n| \leq A_n$ for all n. Then $\int_{-T}^{T}\Big|\sum_{1\leq n\leq N} a_n e(\lambda_n t)\Big|^2 dt \leq 3\int_{-T}^{T}\Big|\sum_{1\leq n\leq N} A_n e(\lambda_n t)\Big|^2 dt$.

The constant 3 has furthermore been shown to be optimal in [Logan, 1988] where the reader will find an intensive discussion on this question. The next lower estimate is also proved there:

Theorem

Let $\lambda_1, \cdots, \lambda_N$ be N be real numbers, and suppose that $a_n \geq 0$ for all n. Then $\int_{-T}^{T} \Big| \sum_{1 \leq n \leq N} a_n e(\lambda_n t) \Big|^2 dt \geq T \sum_{n \leq N} a_n^2$.

We follow the idea of Corollary 3 of [Montgomery and Vaughan, 1974] but rely on [Preissmann, 1984] to get the following.

Theorem (2013)

Let $(a_n)_{n\geq 1}$ be a series of complex numbers that are such that $\sum_n n|a_n|^2 < \infty$ and $\sum_n |a_n| < \infty$. We have, for $T \geq 0$, $\int_0^T \Big| \sum_{n\geq 1} a_n n^{it} \Big|^2 dt = \sum_{n\leq N} |a_n|^2 \big(T + \mathcal{O}^*(2\pi c_0 (n+1))\big)$, where $c_0 = \sqrt{1 + \frac{2}{3}\sqrt{\frac{6}{5}}}$. Moreover, when a_n is real-valued, the constant $2\pi c_0$ may be reduced to πc_0.

This is Lemma 6.2 from [Ramaré, 2016].

Corollary 6.3 and 6.4 of [Ramaré, 2016] contain explicit versions of a Theorem of [Gallagher, 1970]

Theorem (2013)

Let $(a_n)_{n\geq 1}$ be a series of complex numbers that are such that $\sum_n n|a_n|^2 < \infty$ and $\sum_n |a_n| < \infty$. We have, for $T \geq 0$, $\sum_{q \leq Q} \frac{q}{\varphi(q)} \sum_{\substack{\chi \bmod q, \\ \chi \text{ primitive}}} \int_{-T}^{T} \Big| \sum_n a_n \chi(n) n^{it} \Big|^2 dt \leq 7 \sum_n |a_n|^2 (n + Q^2 \max(T, 3))$.

Theorem (2013)

Let $(a_n)_{n\geq 1}$ be a series of complex numbers that are such that $\sum_n n|a_n|^2 < \infty$ and $\sum_n |a_n| < \infty$. We have, for $T \geq 0$, $\sum_{q \leq Q} \frac{q}{\varphi(q)} \sum_{\substack{\chi \bmod q, \\ \chi \text{ primitive}}} \int_{-T}^{T} \Big| \sum_n a_n \chi(n) n^{it} \Big|^2 dt \leq \sum_n |a_n|^2 (43n + \frac{33}{8} Q^2 \max(T, 70))$.

19

Bounds on the Dedekind zeta-function

19.1 1. Size

The knowledge on the general Dedekind zeta is less accomplished than the one of the Riemann zeta-function, but we still have interesting results. Theorem 4 of [Rademacher, 1959] gives the convexity bound. See also section 4.1 of [Trudgian, 2014].

Theorem (1959)

In the strip $-\eta \leq \sigma \leq 1+\eta$, $0 < \eta \leq 1/2$, the Dedekind zeta function $\zeta_K(s)$ belonging to the algebraic number field K of degree n and discriminant d satisfies the inequality $|\zeta_K(s)| \leq 3\left|\frac{1+s}{1-s}\right|\left(\frac{|d||1+s|}{2\pi}\right)^{\frac{1+\eta-\sigma}{2}}\zeta(1+\eta)^n$.

19.2 2. Number of ideals

An explicit approximation of the number of ideals in a number field was given in the PhD memoir [Sunley, 1973] of J.S. Sunley. It is recalled in Theorem 1.1 of [Lee, 2023] and further refined there in Theorem 1.2.

Theorem (2022)

For $x > 0$ and $n_K \geq 0$, the number $I_K(x)$ of integral ideals of norm at most x in the number field K of degree n_K and discriminant Δ_K is approximated given by $I_K(x) = \kappa_K x + \mathcal{O}^*\left(C(K)|\Delta_K|^{\frac{1}{n_K+1}}(\log|\Delta_K|)^{n_K-1}x^{1-\frac{2}{n_K+1}}\right)$ where $C(K) = 0.17\frac{6n_K-2}{n_K-1}2.26^{n_K}e^{4n_K+(26/n_K)}n_K^{n_K+(1/2)}\left(44.39\left(\frac{82}{1000}\right)^{n_K}n_K!+\frac{13}{n_K-1}\right)$.

For $n_K = 2$, the constant Δ_K is about $8.80\cdot10^{11}$. The constant arising from Sunley's work was about $1.75\cdot10^{30}$.

For $n_K = 3$, the constant $C(K)$ is approximately equal to $8.45\cdot10^{11}$. The constant arising from Sunley's work was about $8.57\cdot10^{44}$.

An approximation not relying on the discriminant but on the regulator and the class number has been given in Corollary 2 of [Debaene, 2019].

Theorem (2019)

The notation being as above, we have $I_K(x) = \kappa_K x + O^*\left(n_K^{10n_K^2}(\mathrm{Reg}_K h_K)^{1/n_K}\right)(1 + \log \mathrm{Reg}_K h_K)^{\frac{(n_K-1)^2}{n_K}} x^{1-\frac{1}{n_K}}$.

The reader will also find there an explicit bound of similar strength on the number ideals in a given ideal class. The number of integral ideals in a given ray class is approximated explicitely following the same scheme in Theorem 1 of [Gun *et al.*, 2023].

19.3 3. Bounds for the residue of the Dedekind zeta-function

Let K be a number field over $\mathbb{Q}$ with degree n_K and discriminant Δ_K. Furthermore, suppose that the residue of the Dedekind zeta function $\zeta_K(s)$ at $s = 1$ is denoted κ_K. Unconditional bounds for the residue of the Dedekind zeta-function at $s = 1$ are found in [Louboutin, 2000] and in Section 3 of [Garcia and Lee, 2022].

Theorem (2000, 2022)

If $n_K \geq 3$, then $\dfrac{0.0014480}{n_K g(n_K)|\Delta_K|^{1/n_K}} < \kappa_K \leq \left(\dfrac{e \log|\Delta_K|}{2(n_K-1)}\right)^{n_K-1}$, in which $g(n_K) = 1$ if K has a normal tower over $\mathbb{Q}$ and $g(n_K) = n_K!$ otherwise.

If the Generalised Riemann Hypothesis and Dedekind Conjecture (i.e. ζ_K/ζ is entire) are true, then stronger bounds are found in Corollary 2 of [Garcia and Lee, 2022].

Theorem (2022)

Assume that the Generalised Riemann Hypothesis and the Dedekind Conjecture are true. If $n_K \geq 2$, then $\dfrac{e^{-17.81(n_K-1)}}{\log\log|\Delta_K|} \leq \kappa_K \leq e^{17.81(n_K-1)}(\log\log|\Delta_K|)^{n_K-1}$.

19.4 4. Zeroes and zero-free regions

We denote by $N_K(T)$ the number of zeros ρ, of the Dedekind zeta-function of the number field K of degree n and discriminant d_K, zeros that lie in the critical strip $0 < \Re\rho = \sigma < 1$ and such that $|\Im\rho| \leq T$. After a first result in [Kadiri and Ng, 2012], we find in [Trudgian, 2015] the following result.

Theorem (2014)

When $T \geq 1$, we have $N_K(T) = \dfrac{T}{\pi}\log\left(|d_K|\left(\dfrac{T}{2\pi e}\right)^n\right) + O^*(0.316(\log|d_K| + n\log T) + 5.872n + 3.655)$.

This is improved in [Hasanalizade *et al.*, 2021] into:

Theorem (2021)

When $T \geq 1$, we have $N_K(T) = \frac{T}{\pi}\log\left(|d_K|\left(\frac{T}{2\pi e}\right)^n\right) + O^*\left(0.228(\log|d_K| + n\log T) + 23.108n + 4.520\right)$.

In [Kadiri, 2012], a zero-free region is proved.

Theorem (2012)

Let K be a number field of degree n over $\mathbb{Q}$ and of discriminant d_K such that $|d_K| \geq 2$. The associated Dedekind zeta-function ζ_K has no zeros in the region $\sigma \geq 1 - \dfrac{1}{12.55\log|d_K| + n(9.69\log|t| + 3.03) + 58.63}, |t| \geq 1$ and at most one zero in the region $\sigma \geq 1 - \dfrac{1}{12.74\log|d_K|}, |t| \leq 1$. The exceptional zero, if it exists, is simple and real.

This is improved in [Lee, 2021] into:

Theorem (2021)

Let K be a number field of degree n over $\mathbb{Q}$ and of discriminant d_K such that $|d_K| \geq 2$. The associated Dedekind zeta-function ζ_K has no zeros in the region $\sigma \geq 1 - \dfrac{1}{12.2411\log|d_K| + n(9.5347\log|t| + 0.0501) + 2.2692}, |t| \geq 1$ and if d_K is sufficiently large, then there is at most one zero in the region $\sigma \geq 1 - \dfrac{1}{12.4343\log|d_K|}, |t| < 1$. The exceptional zero, if it exists, is simple and real.

See [Ahn and Kwon, 2014] for a result for Hecke L-series.

In [Louboutin, 2017], a zero-free region is proved. Here is slightly simplified version of his result.

Theorem (2017)

Let K be a number field of degree n over $\mathbb{Q}$ and of discriminant d_K such that $|d_K| \geq 8$. The associated Dedekind zeta-function ζ_K has no zeros in the regions $\sigma \geq 1 - \dfrac{1}{1.7\log|d_K|}, |t| \geq \dfrac{1}{4\log|d_K|}$, $\sigma \geq 1 - \dfrac{1}{2\log|d_K|}, |t| \geq \dfrac{1}{2\log|d_K|}$, and $\sigma \geq 1 - \dfrac{1}{1.62\log|d_K|}, t = 0$.

19.5 5. Real Zeroes.

In [Louboutin, 2015], we find the next result.

Theorem (2015)

Let m be a positive integer. Let K be a number field of degree n over $\mathbb{Q}$ and of discriminant d_K such that $|d_K| \geq \exp((5+\sqrt{5})(\sqrt{m+1}-1)^2)$ The associated Dedekind zeta-function ζ_K has at most m real zeroes in $\sigma \geq 1 - \dfrac{(5+\sqrt{5})(\sqrt{m+1}-1)^2}{2\log|d_K|}$.

20 Primitive Roots

As a consequence of [McGown and Trudgian, 2020], we find the next result.

Theorem (2020)

The least primitive root $g(p)$ modulo the prime p satisfies $g(p) \leq p^{5/8}$ when $p \geq 10^{22}$ and $g(p) < \sqrt{p} - 2$ when $p \geq 10^{56}$.

From [McGown *et al.*, 2016], we also have:

Theorem (2016)

Under GRH, the least primitive root $g(p)$ modulo the prime p satisfies $g(p)409$.

Similar investigations concerning primivite roots modulo p^2 are led in [Kerr *et al.*, 2020] and in [Chen, 2022] where the next theorem is proved.

Theorem (2022)

The least primitive root $h(p)$ modulo p^2 satisfies $h(p) \leq p^{0.74}$ for all $p \geq 2$.

21

Primes in number fields

21.1 1. On the number of primes in a number field

21.2 2. On the Chebotarev Density Theorem

21.3 3. On the least prime ideal

22

TME-EMT: Bibliography

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